

## Time Value of

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## After studying Chapter 3, you should be able to:

1. Understand what is meant by "the time value of money."
2. Understand the relationship between present and future value.
3. Describe how the interest rate can be used to adjust the value of cash flows - both forward and backward - to a single point in time.
4. Calculate both the future and present value of: (a) an amount invested today; (b) a stream of equal cash flows (an annuity); and (c) a stream of mixed cash flows.
5. Distinguish between an "ordinary annuity" and an "annuity due."
6. Use interest factor tables and understand how they provide a shortcut to calculating present and future values.
7. Use interest factor tables to find an unknown interest rate or growth rate when the number of time periods and future and present values are known.
8. Build an "amortization schedule" for an installment-style loan.

## The Time Value of Money

- The Interest Rate
- Simple Interest
- Compound Interest
- Amortizing a Loan
- Compounding More Than Once per Year


## The Interest Rate

## Which would you prefer -- \$10,000 today or $\$ 10,000$ in 5 years?

Obviously, \$10,000 today.
You already recognize that there is TIME VALUE TO MONEY!!

## Why TIME?

## Why is TIME such an important element in your decision?

TIME allows you the opportunity to postpone consumption and earn INTEREST.


## Types of Interest

- Simple Interest

Interest paid (earned) on only the original amount, or principal, borrowed (lent).

- Compound Interest

Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).

## Simple Interest Formula

## Formula $\quad \mathrm{Sl}=\mathrm{P}_{0}(\mathrm{i})(\mathrm{n})$

SI: Simple Interest
$\mathrm{P}_{0}$ : Deposit today ( $\mathbf{t = 0}$ )
i: Interest Rate per Period n:Number of Time Periods

## Simple Interest Example

- Assume that you deposit $\$ 1,000$ in an account earning $7 \%$ simple interest for 2 years. What is the accumulated interest at the end of the 2nd year?

$$
\begin{aligned}
\text { - } \mathrm{SI}=\mathrm{P}_{0}(\mathrm{i})(\mathrm{n}) \\
\$ 1,000(.07)(2)
\end{aligned}
$$

$$
=\$ 140
$$

## Simple Interest (FV)

- What is the Future Value (FV) of the deposit?

$$
\begin{gathered}
F V=P_{o}+S I \\
\$ 1,000+\$ 140
\end{gathered}=\$ 1,140
$$

- Future Value is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.


## Simple Interest (PV)

- What is the Present Value (PV) of the previous problem?

The Present Value is simply the $\$ 1,000$ you originally deposited. That is the value today!

- Present Value is the current value of a future amount of money, or a series of payments, evaluated at a given interest


## Why Compound Interest?

Future Value of a Single \$1,000 Deposit

$\square 10 \%$ Simple Interest
$\square 7 \%$ Compound Interest
$\square 10 \%$ Compound Interest

Assume that you deposit \$1,000 at a compound interest rate of $7 \%$ for 2 years.


# Future Value Single Deposit (Formula) 

$$
\begin{aligned}
\mathrm{FV}_{1} & =\mathrm{P}_{0}(1+\mathrm{i})^{1} \quad=\$ 1,000(1.07) \\
& =\$ 1,070
\end{aligned}
$$

## Compound Interest

You earned \$70 interest on your \$1,000 deposit over the first year.

This is the same amount of interest you would earn under simple interest.


## Future Value Single Deposit (Formula)

$$
\begin{array}{r}
F V_{1}=P_{0}(1+i)^{1} \quad=\$ 1,000(1.07) \\
=\$ 1,070
\end{array}
$$

$$
F V_{2}=F V_{1}(1+i)^{1}
$$

$$
=P_{0}(1+i)(1+i)=
$$

$\$ 1,000(1.07)(1.07) \quad=P_{0}(1+i)^{2}$
= \$1,000(1.07) ${ }^{2}$
= \$1,144.90
${ }_{3-14}$ You earned an EXTRA $\$ 4.90$ in Year 2 with nomnnind nunr cimnln intornct


## General Future Value Formula

$$
\begin{aligned}
& F V_{1}=P_{0}(1+i)^{1} \\
& F V_{2}=P_{0}(1+i)^{2}
\end{aligned}
$$

etc.

## General Future Value Formula:

$$
\begin{aligned}
& F V_{n}=P_{0}(1+i)^{n} \\
\text { or } & F V_{n}=P_{0}\left(F V I F_{i, n}\right) \text {-- See Table I }
\end{aligned}
$$

## Valuation Using Table I

## FVIF $_{i, n}$ is found on Table I at the end of the book.

## Using Future Value Tables

$$
\begin{aligned}
& F V_{2}=\$ 1,000\left(\text { FVIF }_{7 \%, 2}\right) \\
& \$ 1,000(1.145)=\$ 1,145
\end{aligned}
$$

[Due to Rounding]

| Period | $\mathbf{6 \%}$ | $\mathbf{7 \%}$ | $\mathbf{8 \%}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.060 | 1.070 | 1.080 |
| 2 | 1.124 | 1.145 | 1.166 |
| 3 | 1.191 | 1.225 | 1.260 |
| 4 | 1.262 | 1.311 | 1.360 |
| 5 | 1.338 | 1.403 | 1.469 |



## TVM on the Calculator



- Use the highlighted row of keys for solving any of the FV, PV, FVA, PVA, FVAD, and PVAD problems

N: Number of periods
I/Y: Interest rate per period
PV: Present value
PMT: Payment per period
FV: Future value
CLR TVM: Clears all of the inputs into the above TVM keys

## Using The TI BAll+ Calculator

## Inputs

N
I/Y
PV

## PMT

FV

## Compute


— Focus on $3^{\text {rd }}$ Row of keys (will be displayed in slides as shown above)

## Entering the FV Problem



## Press:



## CLR TVM

| 2 |
| :---: |
| 7 |

N
IIY
-1000
PV
PMT
CPT
FV


## Solving the FV Problem

Inputs


N: 2 Periods (enter as 2)
I/Y:7\% interest rate per period (enter as 7 NOT .07)
PV: $\quad \$ 1,000$ (enter as negative as you have "less")
PMT: Not relevant in this situation (enter as 0 )
FV: Compute (Resulting answer is positive)

## Story Problem Example

Julie Miller wants to know how large her deposit of $\$ 10,000$ today will become at a compound annual interest rate of $10 \%$ for 5 years.

\$10,000


## Story Problem Solution

- Calculation based on general formula:

$$
\begin{array}{cc}
F V_{n}=P_{0}(1+i)^{n} & F V_{5}= \\
\$ 10,000(1+0.10)^{5} & =\$ 16,105.10
\end{array}
$$

- Calculation based on Table I:

Rounding]

## Entering the FV Problem



Press:


## Solving the FV Problem

Inputs


## Compute <br> 16,105.10

The result indicates that a $\$ 10,000$ investment that earns $10 \%$ annually for 5 years will result in a future value of $\$ 16,105.10$.

## Double Your Money!!!

# Quick! How long does it take to double $\$ 5,000$ at a compound rate of $12 \%$ per year (approx.)? 

## We will use the "Rule-of-72".



## The "Rule-of-72"

# Quick! How long does it take to double $\$ 5,000$ at a compound rate of $12 \%$ per year (approx.)? 

## Approx. Years to Double $=72$ I $\%$

$$
72 \text { / 12\% = } 6 \text { Years }
$$

[Actual Time is 6.12 Years]

## Solving the Period Problem

Inputs
$12-1,000$
0

$+$| $+2,000$ |  |
| :---: | :---: | :---: |
| $\mathbf{N}$ | I/Y PV PMT FV |

Compute 6.12 years
The result indicates that a $\$ 1,000$ investment that earns $12 \%$ annually will double to $\$ 2,000$ in 6.12 years. Note: 72/12\% = approx. 6 years


## Present Value

## Single Deposit (Graphic)

Assume that you need \$1,000 in 2 years. Let's examine the process to determine how much you need to deposit today at a discount rate of $7 \%$ compounded annually.


## Present Value Single Deposit (Formula)

$$
\begin{aligned}
& \mathrm{PV}_{0}=\mathrm{FV}_{2} /(1+\mathrm{i})^{2}=\$ 1,000 /(1.07)^{2} \\
& =\mathrm{FV}_{2} /(1+\mathrm{i})^{2}=\$ 873.44
\end{aligned}
$$




General Present Value Formula

$$
\begin{aligned}
& \mathrm{PV}_{0}=\mathrm{FV}_{1} /(1+\mathrm{i})^{1} \\
& \mathrm{PV}_{0}=\underset{\mathrm{FV}}{2} /(1+\mathrm{i})^{2} \\
& \text { etc. }
\end{aligned}
$$

## General Present Value Formula:

$$
\begin{aligned}
& \mathrm{PV}_{0}= \\
\text { or } & \mathrm{PV}_{\mathrm{n}} /
\end{aligned}=\mathrm{FV}(1+\mathrm{i})_{\mathrm{n}}\left(\mathrm{PVIF}_{\mathrm{i}, \mathrm{n}}\right) \text {-- See Table II }
$$

## Valuation Using Table II

## PVIF $_{\mathrm{i}, \mathrm{n}}$ is found on Table II at the end of the book.

## Using Present Value Tables

$$
\begin{aligned}
& \mathrm{PV}_{2}=\$ 1,000\left(\mathrm{PVIF}_{7 \%, 2}\right)= \\
& \$ 1,000(.873) \\
& \text { to Rounding] }
\end{aligned}=
$$

| Period | $\mathbf{6 \%}$ | $\mathbf{7 \%}$ | $\mathbf{8 \%}$ |
| :---: | :---: | :---: | :---: |
| 1 | .943 | .935 | .926 |
| 2 | .890 | .873 | .857 |
| 3 | .840 | .816 | .794 |
| 4 | .792 | .763 | .735 |
| 5 | .747 | .713 | .681 |

## Solving the PV Problem

Inputs

2


N

I/Y

## PV PMT

FV

## Compute

-873.44

N: 2 Periods (enter as 2)
I/Y:7\% interest rate per period (enter as 7 NOT .07)
PV: Compute (Resulting answer is negative "deposit")
PMT: Not relevant in this situation (enter as 0 )
FV: \$1,000 (enter as positive as you "receive \$")

## Story Problem Example

Julie Miller wants to know how large of a deposit to make so that the money will grow to $\$ 10,000$ in 5 years at a discount rate of $10 \%$.


## Story Problem Solution

- Calculation based on general formula:

$$
\begin{aligned}
& P V_{0}=F V_{n} I(1+i)^{n} \\
& \$ 10,000 /(1+0.10)^{5}
\end{aligned}
$$

$$
\mathrm{PV}_{0}=
$$

$$
=\$ 6,209.21
$$

- Calculation based on Table I:

$$
=\$ 10,000\left(\text { PVIF }_{10 \%, 5}\right)=\$ 10,000
$$

(.621)

$$
\stackrel{\text { [Due to }}{=} \$ 6,210.00
$$

Rounding]

## Solving the PV Problem

Inputs

510

## $+10,000$

N
I/Y
PV
PMT
FV
-6,209.21

The result indicates that a $\$ 10,000$ future value that will earn 10\% annually for 5 years requires a $\$ 6,209.21$ deposit today (present value).


## Types of Annuities

- An Annuity represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.
- Ordinary Annuity: Payments or receipts occur at the end of each period.
- Annuity Due: Payments or receipts occur at the beginning of each period.



## Examples of Annuities

- Student Loan Payments
- Car Loan Payments
- Insurance Premiums
- Mortgage Payments
- Retirement Savings



## Parts of an Annuity

(Ordinary Annuity)



## Parts of an Annuity

(Annuity Due)
Beginning of Beginning of Beginning of Period 1

Period 2 Period 3

$\$ 100 \quad \$ 100$
\$100
Today


Cash flows occur at the end of the period


$$
\begin{aligned}
& F_{V A}=R(1+i)^{n-1}+R(1+i)^{n-2}+ \\
& \ldots+R(1+i)^{1}+R(1+i)^{0}
\end{aligned}
$$



Cash flows occur at the end of the period

## 0 4


$\mathrm{FVA}_{3}=\$ 1,000(1.07)^{2}+$ $\$ 1,000(1.07)^{1}+\$ 1,000(1.07)^{0}$
$\$ 3,215=$ FVA $_{3}$
= \$1,145 + \$1,070 + \$1,000
= $\$ 3,215$

## Hint on Annuity Valuation

The future value of an ordinary annuity can be viewed as occurring at the end of the last cash flow period, whereas the future value of an annuity due can be viewed as occurring at the beginning of the last cash flow period.

## Valuation Using Table III

$$
\begin{array}{l|l|l}
\mathrm{FVA}_{n}=R\left(\mathrm{FVIFA}_{\mathrm{i} \%, \mathrm{n}}\right) & \mathrm{FVA}_{3} \\
=\$ 1,000\left(\mathrm{FVIFA}_{7 \%, 3}\right) & = \\
\$ 1,000(3.215)=\$ 3,215 & \\
\text { Period } & \mathbf{6 \%} & 7 \% \\
\hline
\end{array}
$$

## Solving the FVA Problem

Inputs


## Compute 3,214.90

N: 3 Periods (enter as 3 year-end deposits) I/Y:7\% interest rate per period (enter as 7 NOT .07) PV: Not relevant in this situation (no beg value) PMT: \$1,000 (negative as you deposit annually) FV: Compute (Resulting answer is positive)

##  <br> Overview View of an Annuity Due -- FVAD

Cash flows occur at the beginning of the period



Cash flows occur at the beginning of the period


## Valuation Using Table III

$$
\begin{aligned}
& \text { FVAD }_{n}=R\left(\text { FVIFA }_{i v, n}\right)(1+i) \\
& \text { FVAD }_{3}=\$ 1,000\left(\text { FVVF }_{7 \%}\right)(1.07) \\
& =\$ 1,000(3.215)(1.07)=\$ 3,440
\end{aligned}
$$

| Period | $\mathbf{6 \%}$ | $\mathbf{7 \%}$ | $\mathbf{8 \%}$ |
| :---: | :---: | :---: | :---: |
| 1 | 1.000 | 1.000 | 1.000 |
| 2 | 2.060 | 2.070 | 2.080 |
| 3 | 3.184 | 3.215 | 3.246 |
| 4 | 4.375 | 4.440 | 4.506 |
| 5 | 5.637 | 5.751 | 5.867 |

## Solving the FVAD Problem

## Inputs


-1,000
PMT
FV

## Compute

 3,439.94Complete the problem the same as an "ordinary annuity" problem, except you must change the calculator setting to "BGN" first. Don't forget to change back!
Step 1: Press $2^{\text {nd }}$ BGN keys
Step 2: Press $2^{\text {nd }} S E T$ keys Step 3: Press $2^{\text {nd }}$ QUIT $\quad$ keys


Cash flows occur at the end of the period

$P^{P V A}$

$$
\begin{aligned}
P V V A_{n} & =R /(1+i)^{1}+R /(1+i)^{2} \\
& +\ldots+R /(1+i)^{n}
\end{aligned}
$$



Cash flows occur at the end of the period

$\$ 2,624.32=$ PVA $_{3}$
$\$ 1,000 /(1.07)^{2}+$
\$1,000/(1.07) ${ }^{3}$
= \$934.58 + \$873.44 + \$816.30
$=\$ 2,624.32$

## Hint on Annuity Valuation

The present value of an ordinary annuity can be viewed as occurring at the beginning of the first cash flow period, whereas the future value of an annuity due can be viewed as occurring at the end of the first cash flow period.

## Valuation Using Table IV

$$
\begin{aligned}
& \text { PVA }_{n}=R\left(\text { PVIFA }_{\mathrm{i} \%, \mathrm{n}}\right) \\
& =\$ 1,000\left(\text { PVIFA }_{7 \%, 3}\right) \\
& \$ 1,000 \text { (2.624) = \$2,624 } \\
& \mathrm{PVA}_{3} \\
& \text { = }
\end{aligned}
$$



## Solving the PVA Problem

Inputs


## Compute

N: 3 Periods (enter as 3 year-end deposits)
I/Y:7\% interest rate per period (enter as 7 NOT .07)
PV: Compute (Resulting answer is positive)
PMT: \$1,000 (negative as you deposit annually)
FV: Not relevant in this situation (no ending value)


## Overview of an Annuity Due -- PVAD

Cash flows occur at the beginning of the period


PVAD
n


Cash Flow

$$
\begin{aligned}
\operatorname{PVAD}_{n} & =R /(1+i)^{0}+R /(1+i)^{1}+\ldots+R /(1+i)^{n-1} \\
& =\operatorname{PVA}_{n}(1+i)
\end{aligned}
$$

3-56


Cash flows occur at the beginning of the period


3


PVAD $_{n}=\$ 1,000 /(1.07)^{0}+\$ 1,000 /(1.07)^{1}+$ $\$ 1,000 /(1.07)^{2}=\$ 2,808.02$

## Valuation Using Table IV

> PVAD $_{n}=R\left(\right.$ PVIFA $\left._{i \%}\right)(1+\mathbf{1 + i})$ PVAD $_{3}=\$ 1,000\left(\mathrm{PVIFA}_{7 \%, 3}\right)(\mathbf{1 . 0 7 )}$ $=\$ 1,000(2.624)(1.07)=\$ 2,808$

| Period | $\mathbf{6 \%}$ | $\mathbf{7 \%}$ | $\mathbf{8 \%}$ |
| :---: | :---: | :---: | :---: |
| 1 | 0.943 | 0.935 | 0.926 |
| 2 | 1.833 | 1.808 | 1.783 |
| 3 | 2.673 | 2.624 | 2.577 |
| 4 | 3.465 | 3.387 | 3.312 |
| 5 | 4.212 | 4.100 | 3.993 |

## Solving the PVAD Problem

Inputs

-1,000
0
$\mathbf{N} \quad \mathrm{I} / \mathbf{Y}$

## Compute

2,808.02
Complete the problem the same as an "ordinary annuity" problem, except you must change the calculator setting to "BGN" first. Don't forget to change back!
Step 1: Press $2^{\text {nd }} B G N \quad$ keys
Step 2: Press $2^{\text {nd }}$ SET keys Step 3: Press $2^{\text {nd }}$ QUIT $\quad$ keys


## Steps to Solve Time Value of Money Problems

1. Read problem thoroughly
2. Create a time line
3. Put cash flows and arrows on time line
4. Determine if it is a PV or FV problem
5. Determine if solution involves a single CF, annuity stream(s), or mixed flow
6. Solve the problem
7. Check with financial calculator (optional)

## Mixed Flows Example

Julie Miller will receive the set of cash flows below. What is the Present Value at a discount rate of $10 \%$.



## How to Solve?

> 1. Solve a "piece-at-a-time" by discounting each piece back to $\mathbf{t}=\mathbf{0}$. 2. Solve a "group-at-a-time" by first breaking problem into groups of annuity streams and any single cash flow groups. Then discount each group back to $\mathbf{t}=\mathbf{0}$.

## "Piece-At-A-Time"

\section*{$\begin{array}{llllll}0 & 1 & 2 & 3 & 4 & 5\end{array}$ | $10 \%$ | $\mid$ | $\mid$ | $\mid$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\$ 600$ | $\$ 600$ | $\$ 400$ | $\$ 400$ | $\$ 100$ |}

$\$ 545.45$ ـ.
\$495.87
\$300.53
\$273.21
\$ 62.09 ․
$\$ 1677.15=\mathrm{PV}_{0}$ of the Mixed
3.63 Flow


## "Group-At-A-Time" (\#2)




## Solving the Mixed Flows Problem using CF Registry



Use the highlighted key for starting the process of solving a mixed cash flow problem

- Press the CF key and down arrow key through a few of the keys as you look at the definitions on the next slide


Solving the Mixed Flows

## Problem using CF Registry

## Defining the calculator variables:

For CFO: This is ALWAYS the cash flow occurring at time $\mathbf{t = 0}$ (usually 0 for these problems)
For Cnn:* This is the cash flow SIZE of the nth group of cash flows. Note that a "group" may only contain a single cash flow (e.g., \$351.76).
For Fnn:*This is the cash flow FREQUENCY of the nth group of cash flows. Note that this is always a positive whole number (e.g., 1, 2, 20, etc.).

* nn represents the nth cash flow or frequency. Thus, the first cash flow is C01, while the tenth cash flow is C10.


# Solving the Mixed Flows Problem using CF Registry 

## Steps in the Process

## Step 1: Press CF <br> 

 Step 2: Press $2^{\text {nd }}$ CLR Work keys Step 3: For CFO Press Step 4: For C01 Press Step 5: For F01 Press Step 6: For C02 Press Step 7: For F02 Press| 0 Enter | Keys |
| :---: | :---: |
| 600 Enter | keys |
| 2 Enter | Keys |
| 400 Enter | keys |
| 2 Enter | keys |



## Solving the Mixed Flows Problem using CF Registry

## Steps in the Process

Step 8: For C03 Press
Step 9: For F03 Press Step 10: Press
Step 11: Press


Enter
 keys

Step 12: For I=, Enter 10 Enter Step 13: Press CPT


Result: Present Value $=\$ 1,677.15$


Frequency of Compounding

## General Formula:

$$
\mathrm{FV}_{\mathrm{n}}=\mathrm{PV} \mathrm{~V}_{0}(1+[\mathrm{i} / \mathrm{m}])^{\mathrm{mn}}
$$

n: Number of Years
Compounding Periods per Year i: Annual Interest Rate $\quad \mathrm{FV}_{\mathrm{n}, \mathrm{m}}$ : FV at the end of Year $n$

$\mathrm{PV}_{0}$ : PV of the Cash Flow today

## Impact of Frequency

Julie Miller has $\$ 1,000$ to invest for 2 Years at an annual interest rate of
12\%.

Annual $\quad \mathrm{FV}_{2}=1,000(1+[.12 / 1])^{(1)(2)}$

$$
=1,254.40
$$

Semi $\quad \mathrm{FV}_{2}=1,000(1+[.12 / 2])^{(2)(2)}$

$$
=1,262.48
$$

## Impact of Frequency

# Qrtly <br> <br> $\mathrm{FV}_{2}=1,000(1+[.12 / 4])^{(4)(2)}$ <br> <br> $\mathrm{FV}_{2}=1,000(1+[.12 / 4])^{(4)(2)}$ <br> $=1,266.77$ 

Monthly $\quad \mathrm{FV}_{2}=1,000(1+[.12 / 12])^{(12)(2)}$

$$
=1,269.73
$$

Daily

$$
\begin{aligned}
& \mathrm{FV}_{2}=1,000(1+[.12 / 365])^{(365)(2)} \\
& =1,271.20
\end{aligned}
$$



The result indicates that a $\$ 1,000$ investment that earns a $12 \%$ annual rate compounded quarterly for 2 years will earn a future value of $\$ 1,266.77$.

## Solving the Frequency Problem (Quarterly Altern.)

## Press:



## Solving the Frequency Problem (Daily)

Inputs

| 2(365) | $12 / 365$ |
| :---: | :---: |
| $\mathbf{N}$ | $\mathbf{I} / \mathbf{Y}$ | -1,000 PV PMT

## Compute

 1271.20The result indicates that a \$1,000 investment that earns a $12 \%$ annual rate compounded daily for 2 years will earn a future value of $\$ 1,271.20$.

## Solving the Frequency Problem (Daily Alternative)

\section*{* Texas Instruments <br> | BRL = |
| :---: |
|  |  |

## Press:




## Effective Annual Interest Rate

## Effective Annual Interest Rate

The actual rate of interest earned (paid) after adjusting the nominal rate for factors such as the number of compounding periods per year.

$$
(1+[\mathrm{i} / \mathrm{m}])^{\mathrm{m}}-1
$$



# BWs Effective Annual Interest Rate 

Basket Wonders (BW) has a \$1,000 CD at the bank. The interest rate is $6 \%$ compounded quarterly for 1 year. What is the Effective Annual Interest Rate (EAR)?
EAR $=(1+6 \% / 4)^{4}-1$
1.0614-1 = . 0614 or $6.14 \%$ !

## Converting to an EAR

## Press:

## $2^{\text {nd }}$ <br> I Conv

## 6 ENTER


$2^{\text {nd }}$
QUIT


## Steps to Amortizing a Loan

1. Calculate the payment per period.
2. Determine the interest in Period t . (Loan Balance at t-1) x (i\% / m)
3. Compute principal payment in Period $\mathbf{t}$. (Payment - Interest from Step 2)
4. Determine ending balance in Period $t$. (Balance - principal payment from Step 3)
5. Start again at Step 2 and repeat.

## Amortizing a Loan Example

Julie Miller is borrowing \$10,000 at a compound annual interest rate of $12 \%$. Amortize the loan if annual payments are made for 5 years.
Step 1: Payment

$$
\begin{aligned}
& \text { PV }_{0}=R\left(\text { PVIFA }_{i \%, n}\right) \\
& \$ 10,000=R\left(\text { PVIFA }_{12 \%, 5}\right) \\
& \$ 10,000=R(3.605) \\
& R=\$ 10,000 / 3.605=\$ 2,774
\end{aligned}
$$

## Amortizing a Loan Example

| End of <br> Year | Payment | Interest | Principal | Ending <br> Balance |
| :---: | ---: | :---: | :---: | ---: |
| 0 | -- | --- | --- | $\$ 10,000$ |
| 1 | $\$ 2,774$ | $\$ 1,200$ | $\$ 1,574$ | 8,426 |
| 2 | 2,774 | 1,011 | 1,763 | 6,663 |
| 3 | 2,774 | 800 | 1,974 | 4,689 |
| 4 | 2,774 | 563 | 2,211 | 2,478 |
| 5 | $\frac{2,775}{}$ | 297 | 2,478 | 0 |

[Last Payment Slightly Higher Due to Rounding]

## Solving for the Payment

$\begin{array}{llll}\text { Inputs } & 5 & 12 & 10,000\end{array}$
$\mathbf{N} \quad \mathrm{I} / \mathrm{Y}$ PV

# PMT 

Compute
-2774.10
The result indicates that a $\$ 10,000$ loan that costs $12 \%$ annually for 5 years and will be completely paid off at that time will require $\$ 2,774.10$ annual payments.


## Using the Amortization Functions of the Calculator



## Press:

## $2^{\text {nd }}$ Amort

## 1 ENTER



Results:
BAL = 8,425.90*
PRN = -1,574.10*
$\downarrow$
INT = -1,200.00*


Year 1 information only

## Using the Amortization Functions of the Calculator



## Press:

## $2^{\text {nd }}$ Amort

## 2 ENTER

2
ENTER
Results:

| BAL $=6,662.91^{*}$ | $\downarrow$ |
| :--- | :--- |
| PRN $=-1,763.99^{*}$ | $\downarrow$ |
| INT $=-1,011.11^{*}$ | $\downarrow$ |

Year 2 information only

## Using the Amortization Functions of the Calculator



## Press:

## $2^{\text {nd }}$ Amort

## 1 ENTER

## 5 ENTER

Results:

| BAL $=$ | 0.00 |
| :--- | :--- |
| PRN $=-10,000.00$ | $\downarrow$ |
| INT $=-3,870.49$ | $\downarrow$ |

Entire 5 Years of loan information (see the total line of 3-82)

## Usefulness of Amortization

1. Determine Interest Expense -Interest expenses may reduce taxable income of the firm.
2. Calculate Debt Outstanding -The quantity of outstanding debt may be used in financing the day-to-day activities of the firm.
