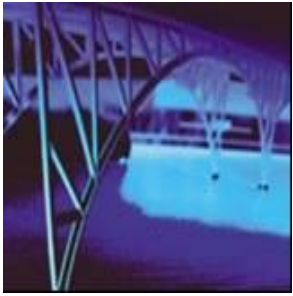


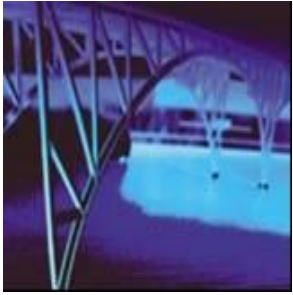
Prof-Parashar Dave
Department-commerce

Time Value of Money



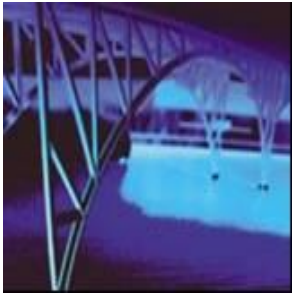
After studying Chapter 3, you should be able to:

- 1. Understand what is meant by "the time value of money."**
- 2. Understand the relationship between present and future value.**
- 3. Describe how the interest rate can be used to adjust the value of cash flows – both forward and backward – to a single point in time.**
- 4. Calculate both the future and present value of: (a) an amount invested today; (b) a stream of equal cash flows (an annuity); and (c) a stream of mixed cash flows.**
- 5. Distinguish between an "ordinary annuity" and an "annuity due."**
- 6. Use interest factor tables and understand how they provide a shortcut to calculating present and future values.**
- 7. Use interest factor tables to find an unknown interest rate or growth rate when the number of time periods and future and present values are known.**
- 8. Build an "amortization schedule" for an installment-style loan.**



The Time Value of Money

- **The Interest Rate**
- **Simple Interest**
- **Compound Interest**
- **Amortizing a Loan**
- **Compounding More Than Once per Year**

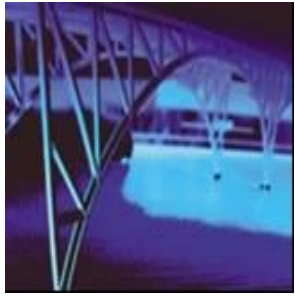


The Interest Rate

Which would you prefer -- \$10,000 today or \$10,000 in 5 years?

Obviously, \$10,000 today.

You already recognize that there is TIME VALUE TO MONEY!!



Why TIME?

Why is **TIME such an important element in your decision?**

TIME allows you the *opportunity* to postpone consumption and earn **INTEREST**.



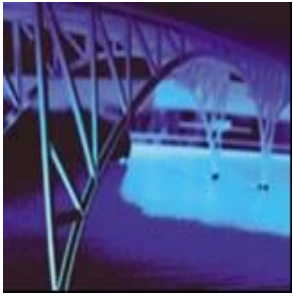
Types of Interest

- **Simple Interest**

Interest paid (earned) on only the original amount, or principal, borrowed (lent).

- **Compound Interest**

Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).



Simple Interest Formula

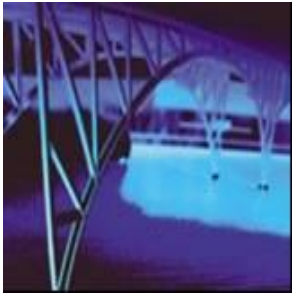
Formula **$SI = P_0(i)(n)$**

SI: Simple Interest

P_0 : Deposit today (t=0)

i : Interest Rate per Period

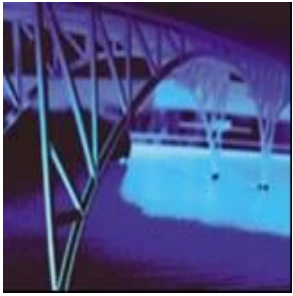
n : Number of Time Periods



Simple Interest Example

- Assume that you deposit **\$1,000** in an account earning **7%** simple interest for **2** years. *What is the accumulated interest at the end of the 2nd year?*

- $$\begin{aligned} SI &= P_0(i)(n) \\ \$1,000(.07)(2) &= \$140 \end{aligned}$$

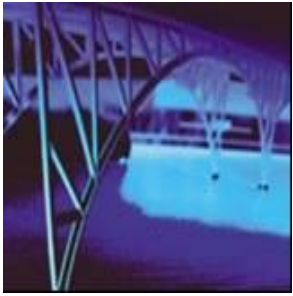


Simple Interest (FV)

- What is the **Future Value (FV)** of the deposit?

$$\begin{aligned} FV &= P_0 + SI \\ \$1,000 + \$140 &= \$1,140 \end{aligned}$$

- Future Value is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.

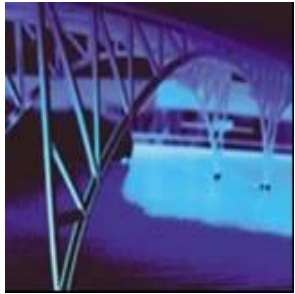


Simple Interest (PV)

- What is the **Present Value (PV)** of the previous problem?

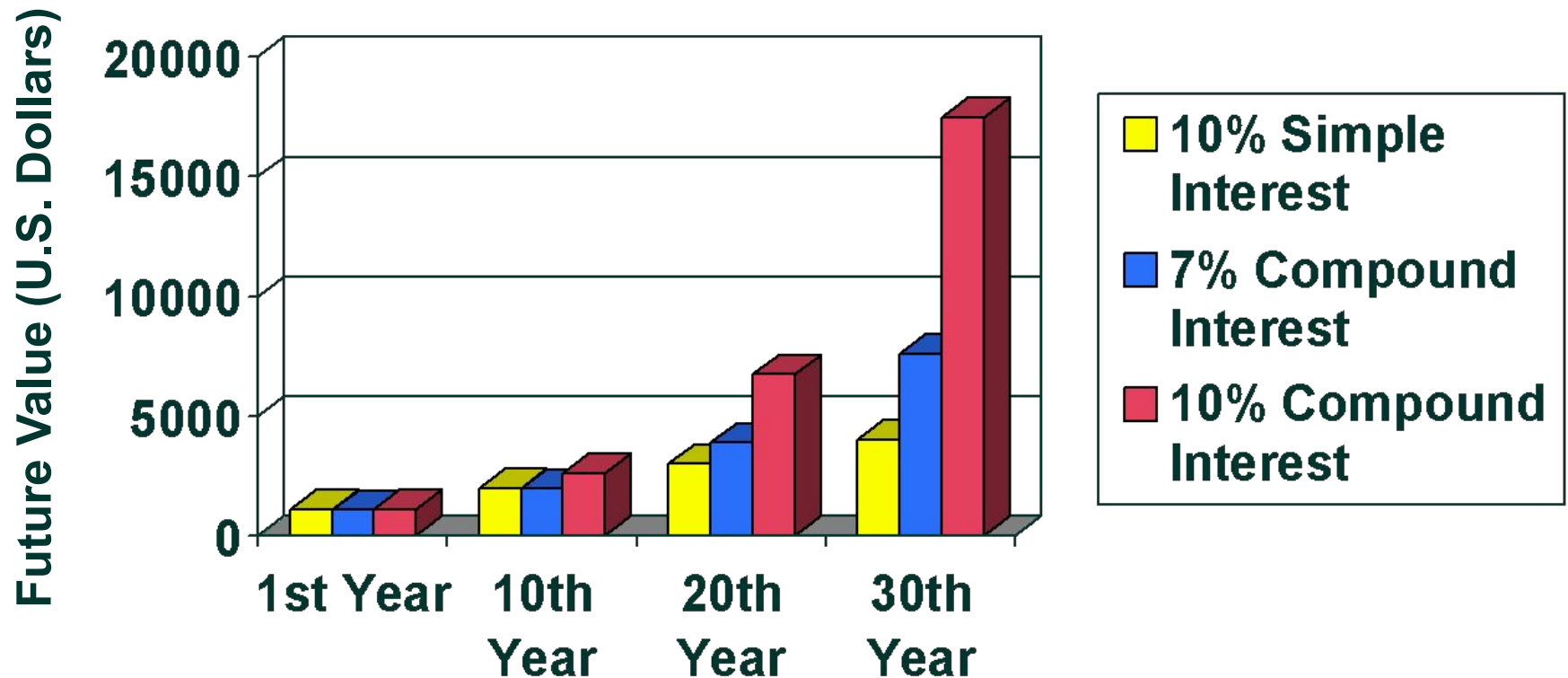
*The **Present Value** is simply the **\$1,000** you originally deposited. That is the value today!*

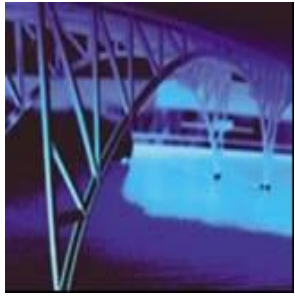
- **Present Value** is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate.



Why Compound Interest?

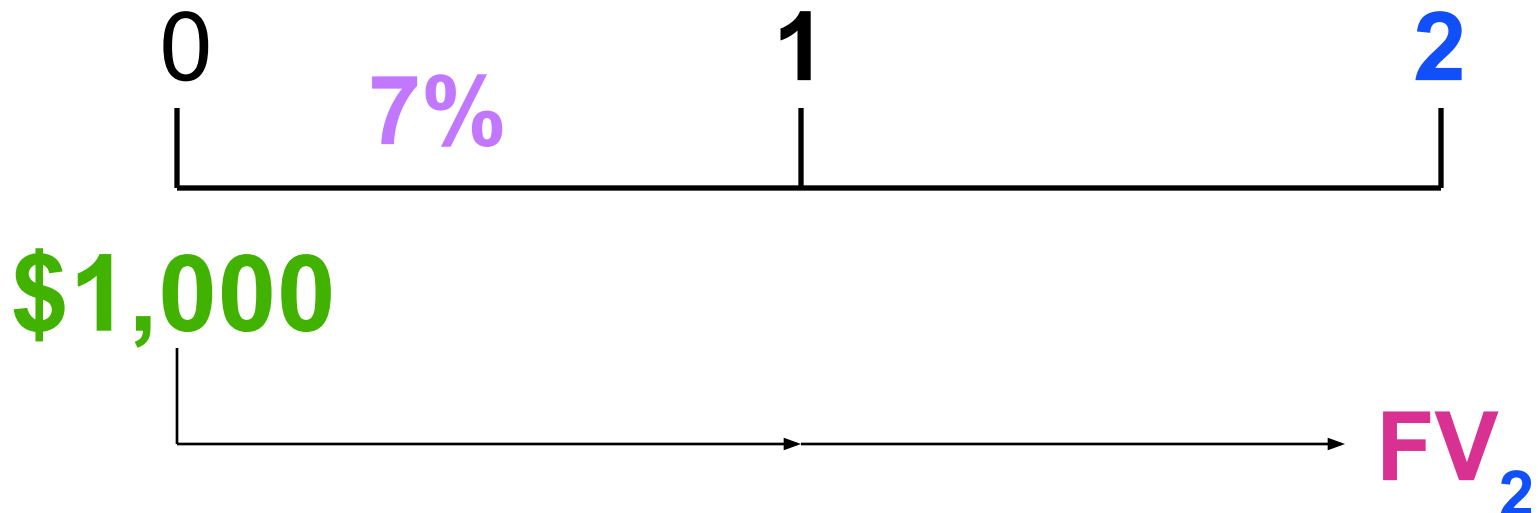
Future Value of a Single \$1,000 Deposit

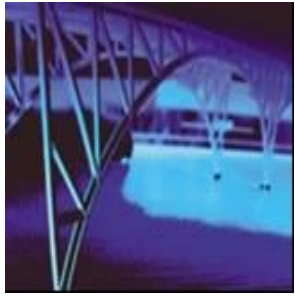




Future Value Single Deposit (Graphic)

Assume that you deposit **\$1,000** at a compound interest rate of **7%** for **2 years**.





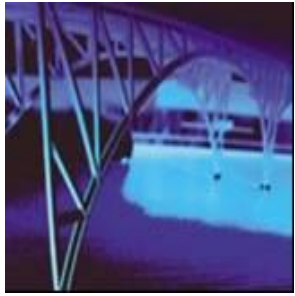
Future Value Single Deposit (Formula)

$$\begin{aligned} FV_1 &= P_0 (1+i)^1 &&= \$1,000 (1.07) \\ &= \$1,070 \end{aligned}$$

Compound Interest

You earned \$70 interest on your \$1,000 deposit over the first year.

This is the same amount of interest you would earn under simple interest.

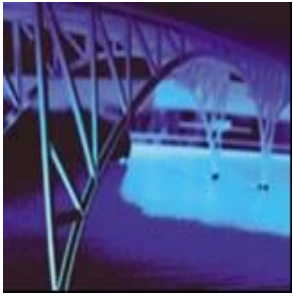


Future Value Single Deposit (Formula)

$$FV_1 = P_0 (1+i)^1 = \$1,000 (1.07) = \$1,070$$

$$\begin{aligned} FV_2 &= FV_1 (1+i)^1 \\ &= P_0 (1+i)(1+i) = \\ & \$1,000(1.07)(1.07) = P_0 (1+i)^2 \\ &= \$1,000(1.07)^2 \\ &= \$1,144.90 \end{aligned}$$

3-14 You earned an **EXTRA \$4.90** in Year 2 with compound over simple interest



General Future Value Formula

$$FV_1 = P_0(1+i)^1$$

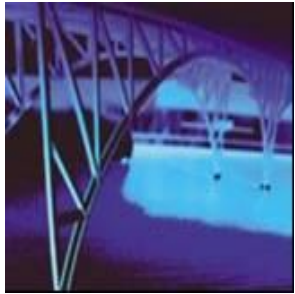
$$FV_2 = P_0(1+i)^2$$

etc.

General Future Value Formula:

$$FV_n = P_0(1+i)^n$$

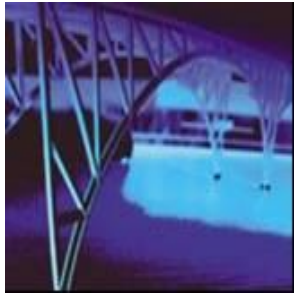
or $FV_n = P_0(FVIF_{i,n})$ -- See Table I



Valuation Using Table I

FVIF_{i,n} is found on Table I
at the end of the book.

Period	6%	7%	8%
1	1.060	1.070	1.080
2	1.124	1.145	1.166
3	1.191	1.225	1.260
4	1.262	1.311	1.360
5	1.338	1.403	1.469



Using Future Value Tables

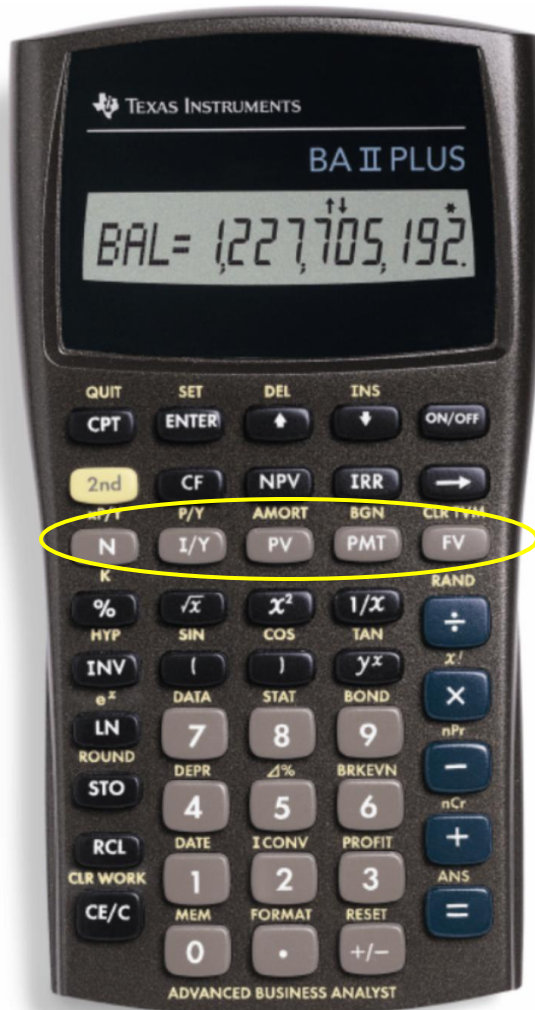
$$\begin{aligned} FV_2 &= \$1,000 (FVIF_{7\%,2}) \\ &= \$1,000 (1.145) \\ &= \$1,145 \end{aligned}$$

[Due to Rounding]

Period	6%	7%	8%
1	1.060	1.070	1.080
2	1.124	1.145	1.166
3	1.191	1.225	1.260
4	1.262	1.311	1.360
5	1.338	1.403	1.469



TVM on the Calculator



- Use the highlighted row of keys for solving any of the FV, PV, FVA, PVA, FVAD, and PVAD problems

N: Number of periods

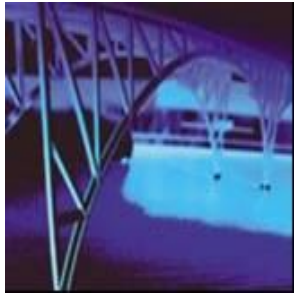
I/Y: Interest rate per period

PV: Present value

PMT: Payment per period

FV: Future value

CLR TVM: Clears all of the inputs into the above TVM keys



Using The TI BAII+ Calculator

Inputs

N

I/Y

PV

PMT

FV

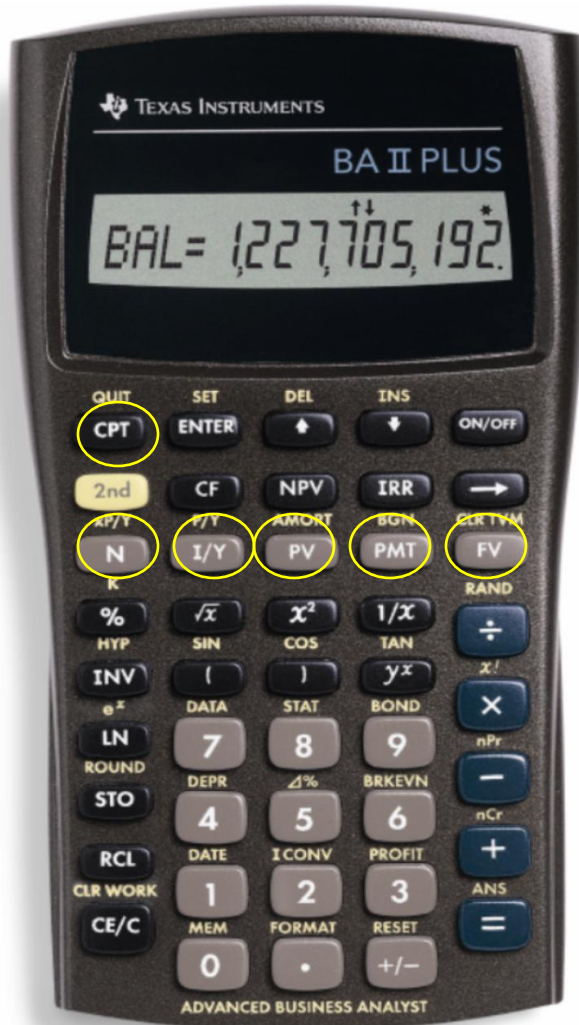
Compute



□ Focus on 3rd Row of keys (will be displayed in slides as shown above)



Entering the FV Problem



Press:

2nd

CLR TVM

2

N

7

I/Y

-1000

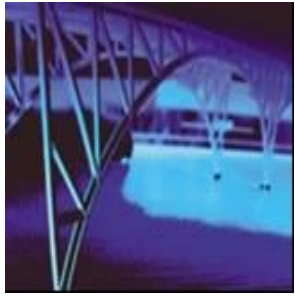
PV

0

PMT

CPT

FV



Solving the FV Problem

Inputs	2	7	-1,000	0	
	N	I/Y	PV	PMT	FV
Compute	1,144.90				

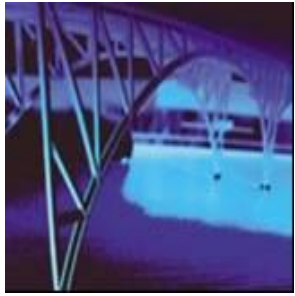
N: 2 Periods (enter as 2)

I/Y: 7% interest rate per period (enter as 7 NOT .07)

PV: \$1,000 (enter as negative as you have “less”)

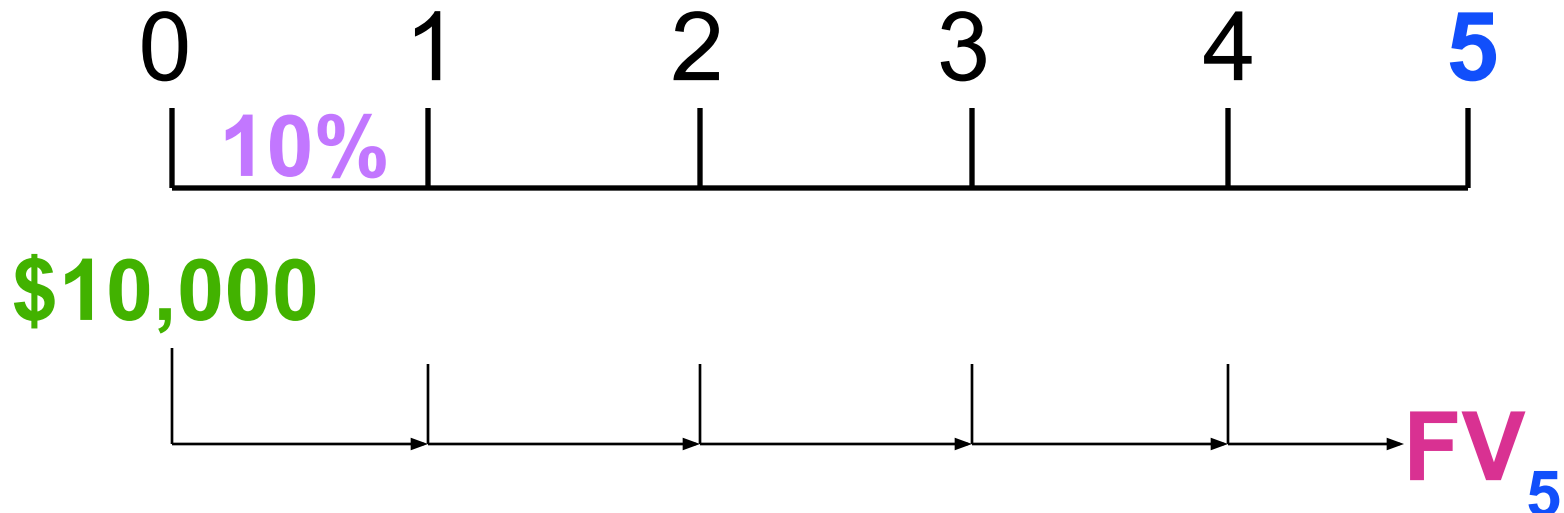
PMT: Not relevant in this situation (enter as 0)

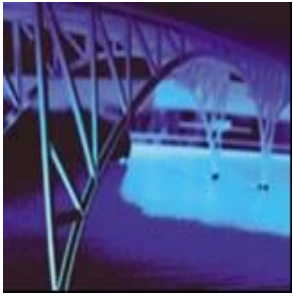
FV: Compute (Resulting answer is positive)



Story Problem Example

Julie Miller wants to know how large her deposit of **\$10,000** today will become at a compound annual interest rate of **10%** for **5 years**.





Story Problem Solution

- Calculation based on general formula:

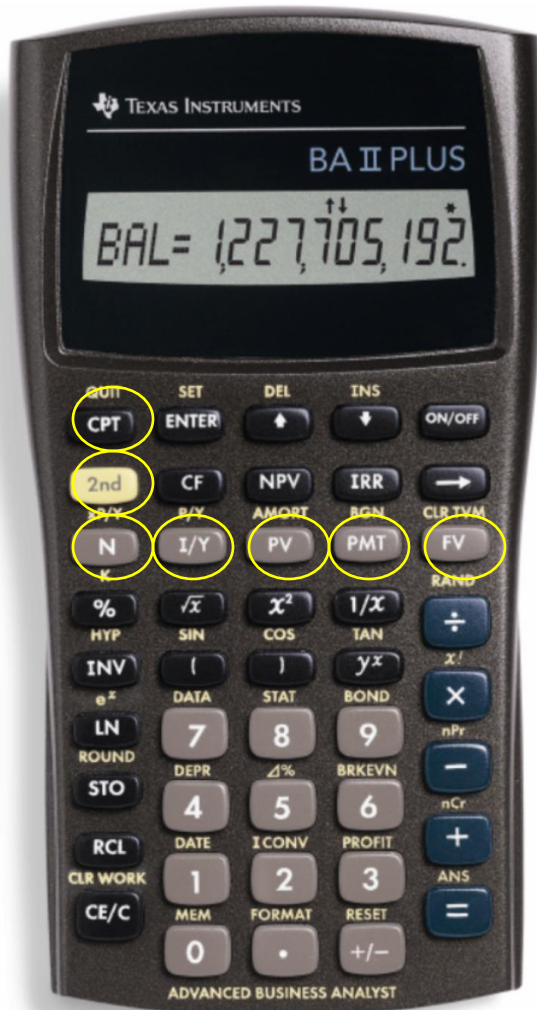
$$\begin{aligned} FV_n &= P_0 (1+i)^n \\ \$10,000 (1+0.10)^5 &= FV_5 = \$16,105.10 \end{aligned}$$

- Calculation based on Table I:

$$\begin{aligned} &= \$10,000 (FVIF_{10\%, 5}) \\ &= \$10,000 (1.611) \\ &= \$16,110 \quad [Due to Rounding] \end{aligned}$$



Entering the FV Problem



Press:

2nd

CLR TVM

5

N

10

I/Y

-10000

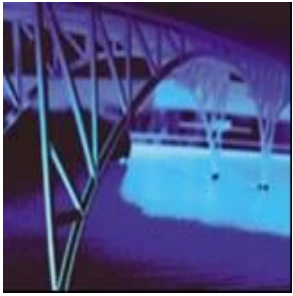
PV

0

PMT

CPT

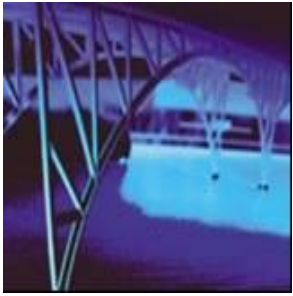
FV



Solving the FV Problem

Inputs	5	10	-10,000	0	
	N	I/Y	PV	PMT	FV
Compute	16,105.10				

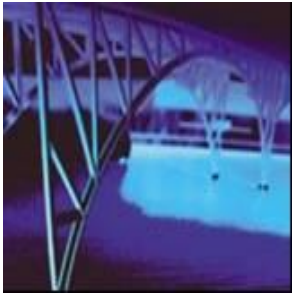
The result indicates that a **\$10,000** investment that earns **10%** annually for **5 years** will result in a future value of **\$16,105.10**.



Double Your Money!!!

Quick! How long does it take to double \$5,000 at a compound rate of 12% per year (approx.)?

We will use the **“Rule-of-72”**.



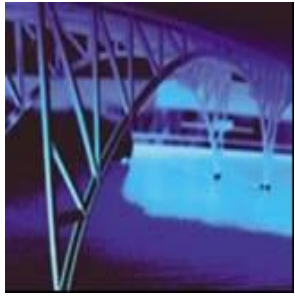
The “Rule-of-72”

Quick! How long does it take to double \$5,000 at a compound rate of 12% per year (approx.)?

Approx. *Years to Double* = $72 / i\%$

$72 / 12\% = \underline{6 \text{ Years}}$

[Actual Time is 6.12 Years]

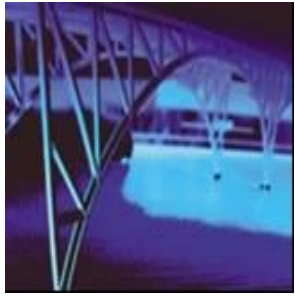


Solving the Period Problem

Inputs	+2,000	12	-1,000	0	
	N	I/Y	PV	PMT	FV
Compute	6.12 years				

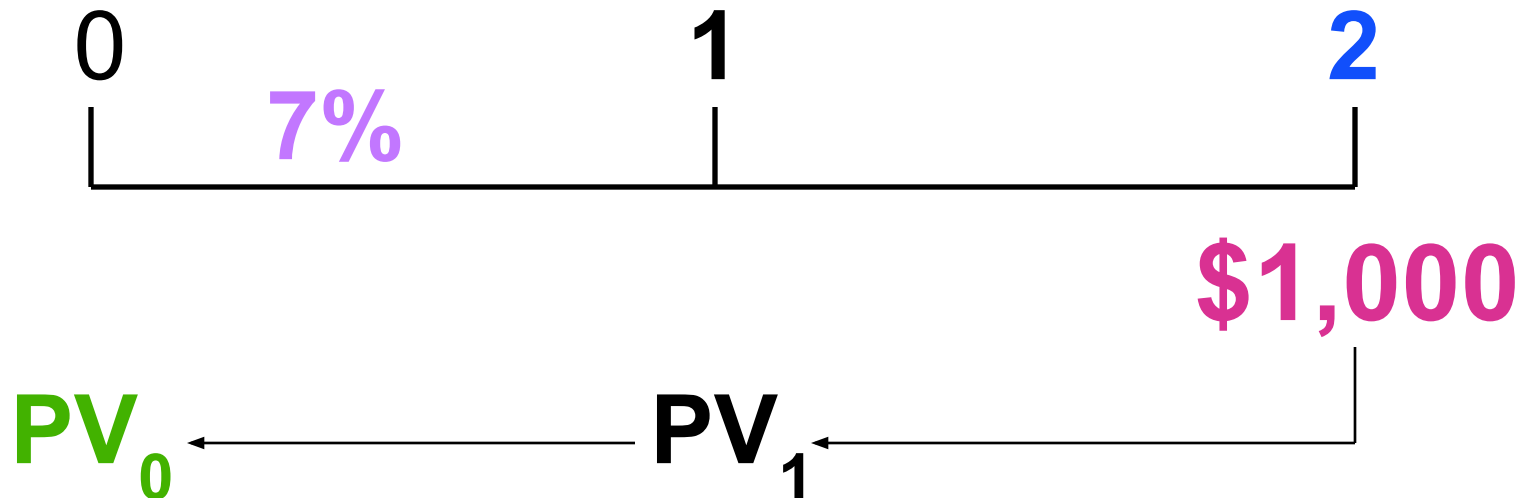
The result indicates that a **\$1,000** investment that earns **12%** annually will double to **\$2,000** in **6.12 years**.

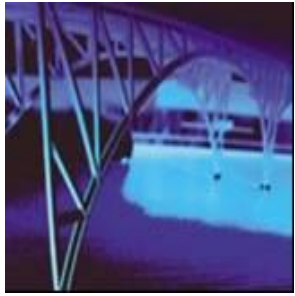
Note: $72/12\% = \text{approx. } 6 \text{ years}$



Present Value Single Deposit (Graphic)

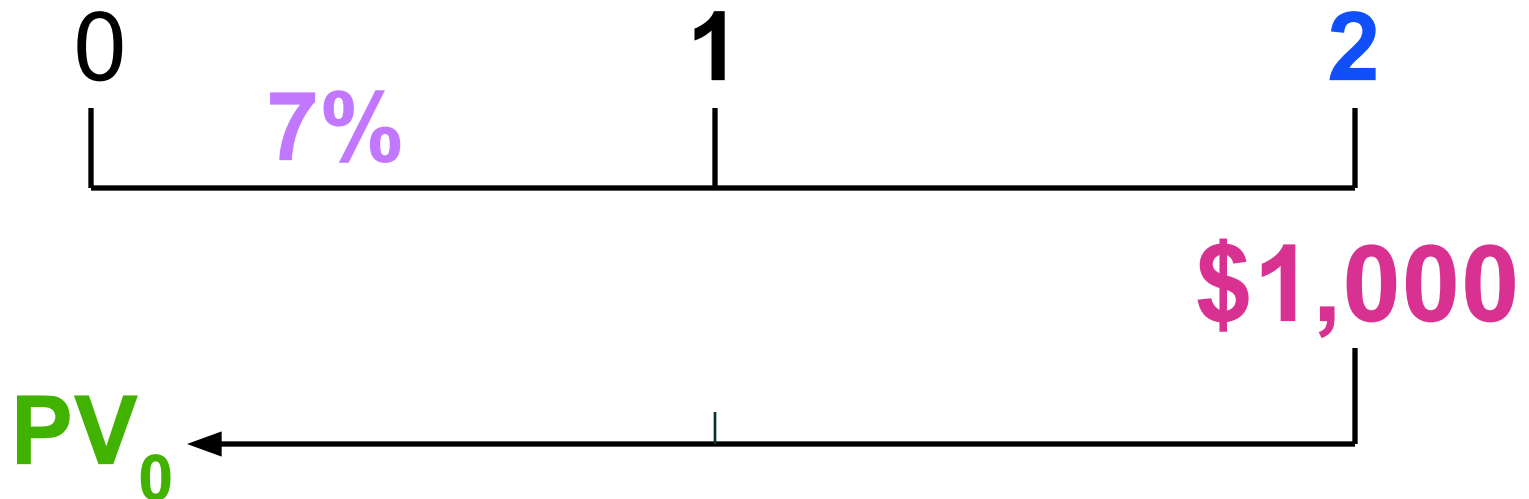
Assume that you need **\$1,000** in **2 years**.
Let's examine the process to determine
how much you need to deposit today at a
discount rate of **7%** compounded annually.

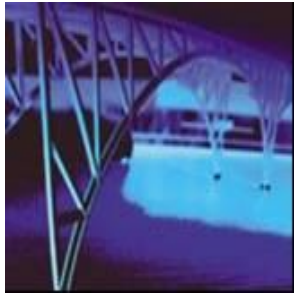




Present Value Single Deposit (Formula)

$$\begin{aligned} PV_0 &= FV_2 / (1+i)^2 = \$1,000 / (1.07)^2 \\ &= FV_2 / (1+i)^2 = \$873.44 \end{aligned}$$





General Present Value Formula

$$PV_0 = FV_1 / (1+i)^1$$

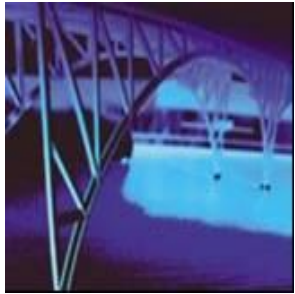
$$PV_0 = FV_2 / (1+i)^2$$

etc.

General Present Value Formula:

$$PV_0 = FV_n / (1+i)^n$$

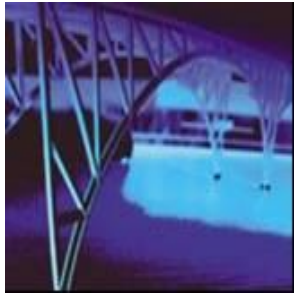
or $PV_0 = FV_n (PVIF_{i,n})$ -- See *Table II*



Valuation Using Table II

PVIF_{*i,n*} is found on Table II
at the end of the book.

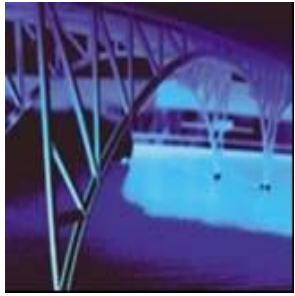
Period	6%	7%	8%
1	.943	.935	.926
2	.890	.873	.857
3	.840	.816	.794
4	.792	.763	.735
5	.747	.713	.681



Using Present Value Tables

$$\begin{aligned} \text{PV}_2 &= \$1,000 (\text{PVIF}_{7\%,2}) \\ \$1,000 (.873) &= \$873 \text{ [Due to Rounding]} \end{aligned}$$

Period	6%	7%	8%
1	.943	.935	.926
2	.890	.873	.857
3	.840	.816	.794
4	.792	.763	.735
5	.747	.713	.681



Solving the PV Problem

Inputs	2	7	0		
	+1,000				
	N	I/Y	PV	PMT	FV
Compute			-873.44		

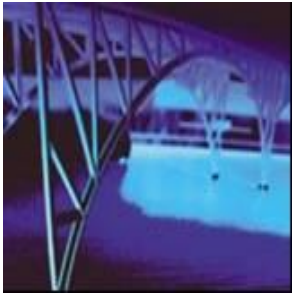
N: 2 Periods (enter as 2)

I/Y: 7% interest rate per period (enter as 7 NOT .07)

PV: Compute (Resulting answer is negative “deposit”)

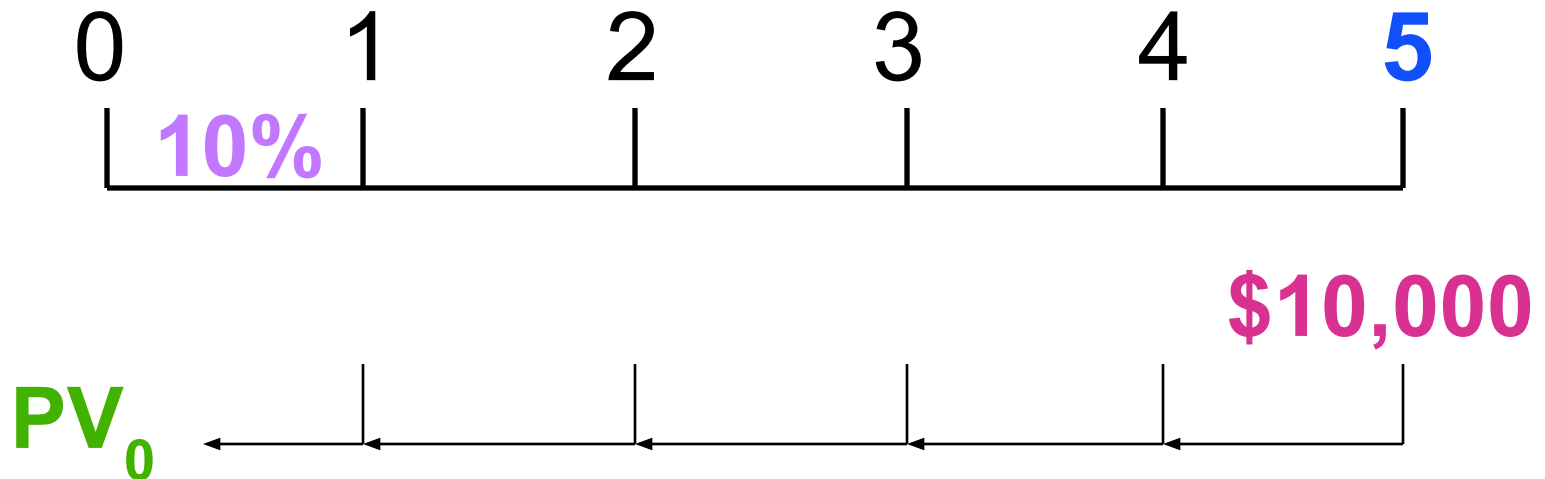
PMT: Not relevant in this situation (enter as 0)

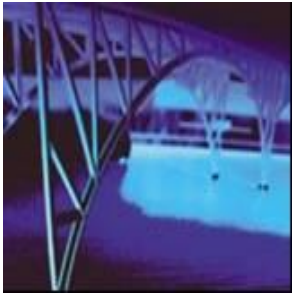
FV: \$1,000 (enter as positive as you “receive \$”)



Story Problem Example

Julie Miller wants to know how large of a deposit to make so that the money will grow to **\$10,000** in **5 years** at a discount rate of **10%**.



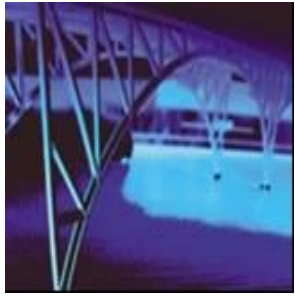


Story Problem Solution

- Calculation based on general formula:

$$\begin{aligned} PV_0 &= FV_n / (1+i)^n \\ \$10,000 &/ (1+0.10)^5 \\ &= \$6,209.21 \end{aligned}$$

- Calculation based on Table I: PV_0
 $= \$10,000 (PVIF_{10\%, 5})$
 $(.621)$
Rounding
 $= \$6,210.00$ [Due to PV_0]



Solving the PV Problem

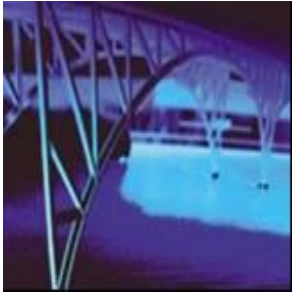
Inputs	5	10	0		
	+10,000				
	N	I/Y	PV	PMT	FV
Compute			-6,209.21		

The result indicates that a **\$10,000** future value that will earn **10%** annually for **5 years** requires a **\$6,209.21** deposit today (present value).



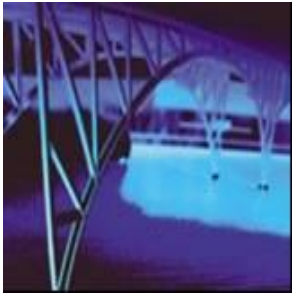
Types of Annuities

- ***An Annuity*** represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.
- **Ordinary Annuity**: Payments or receipts occur at the **end** of each period.
- **Annuity Due**: Payments or receipts occur at the **beginning** of each period.



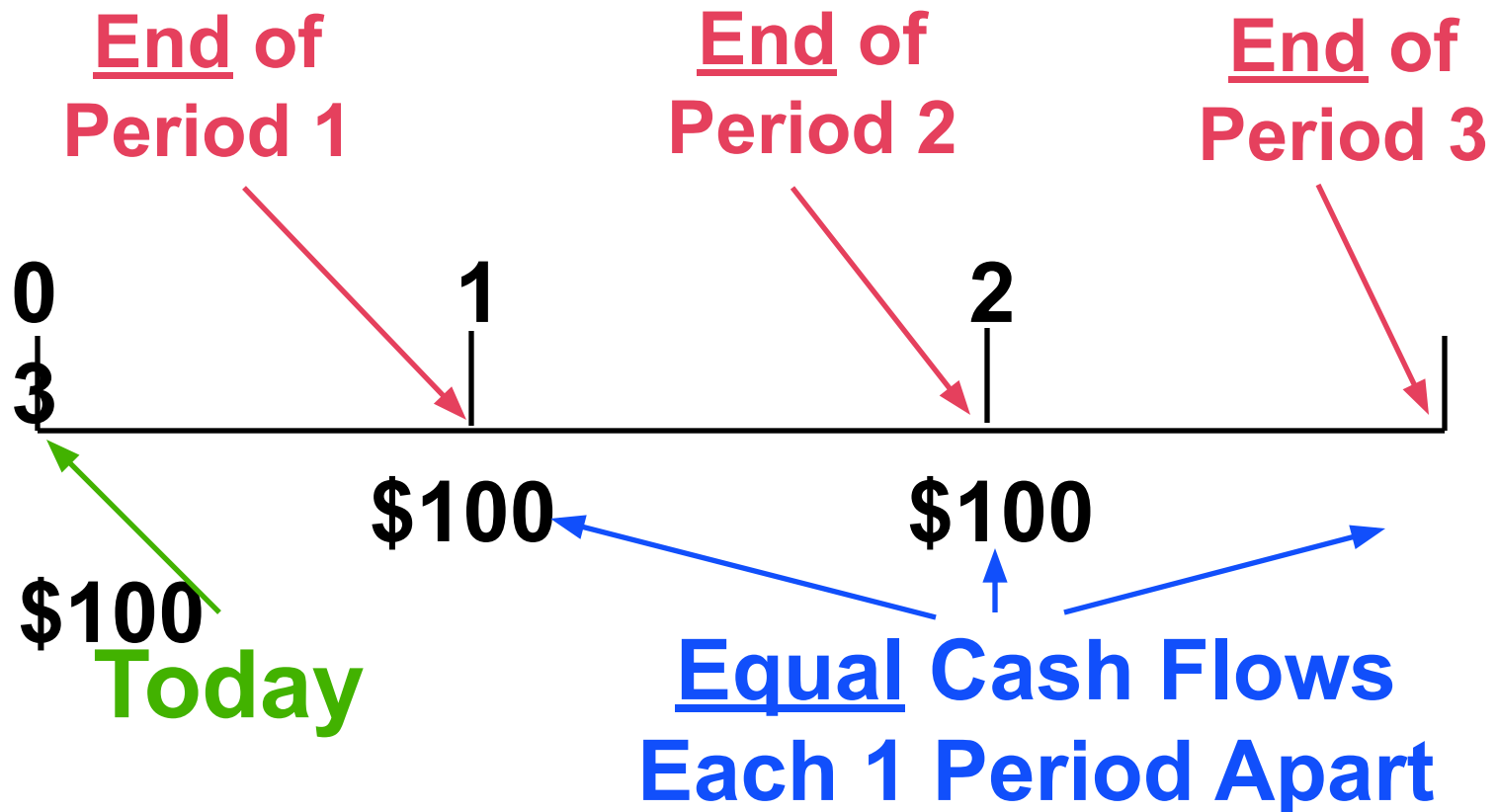
Examples of Annuities

- **Student Loan Payments**
- **Car Loan Payments**
- **Insurance Premiums**
- **Mortgage Payments**
- **Retirement Savings**



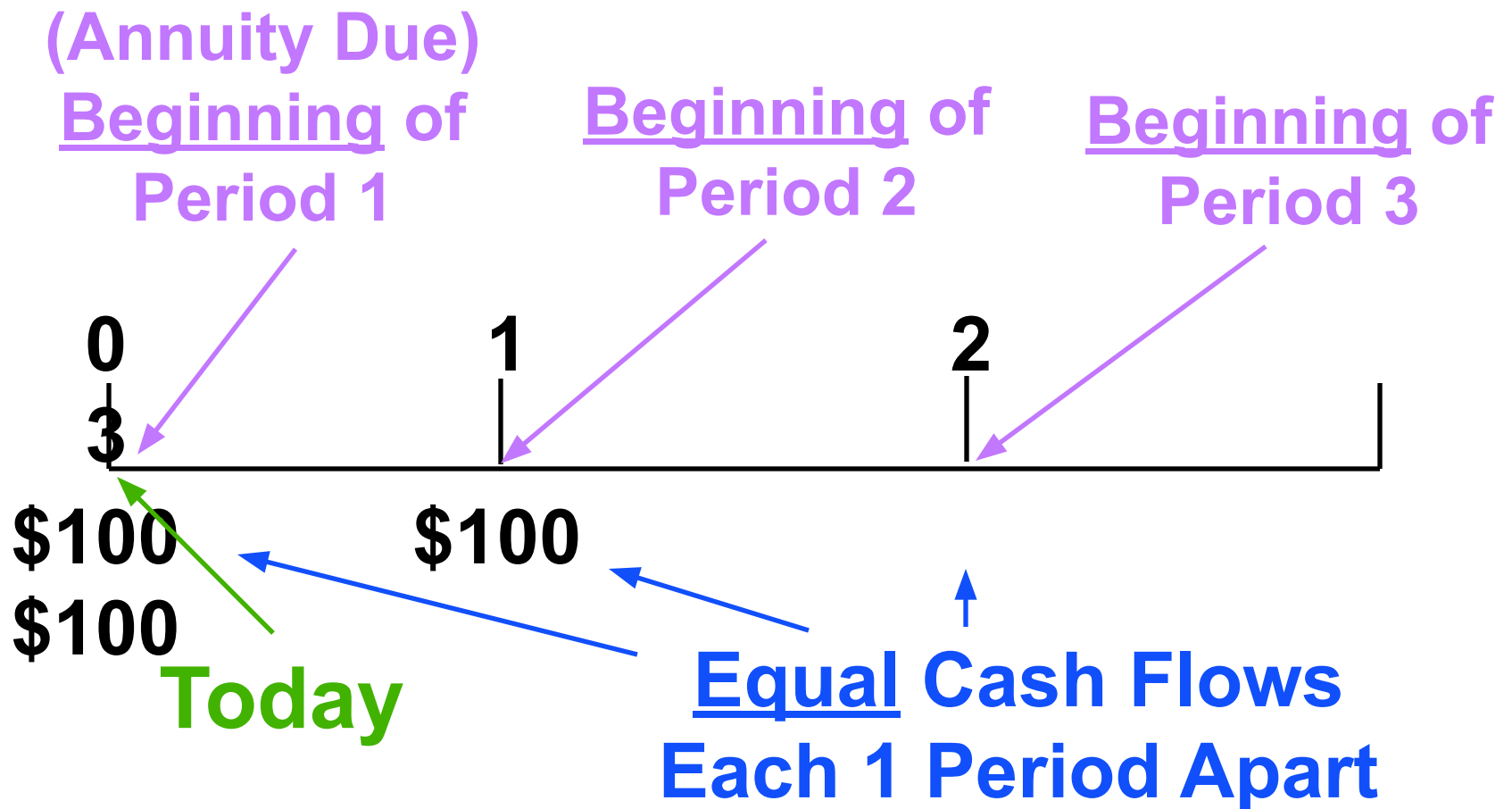
Parts of an Annuity

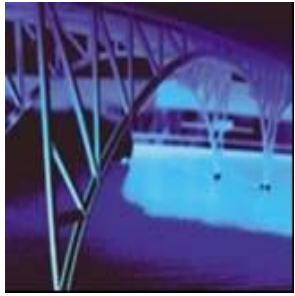
(Ordinary Annuity)





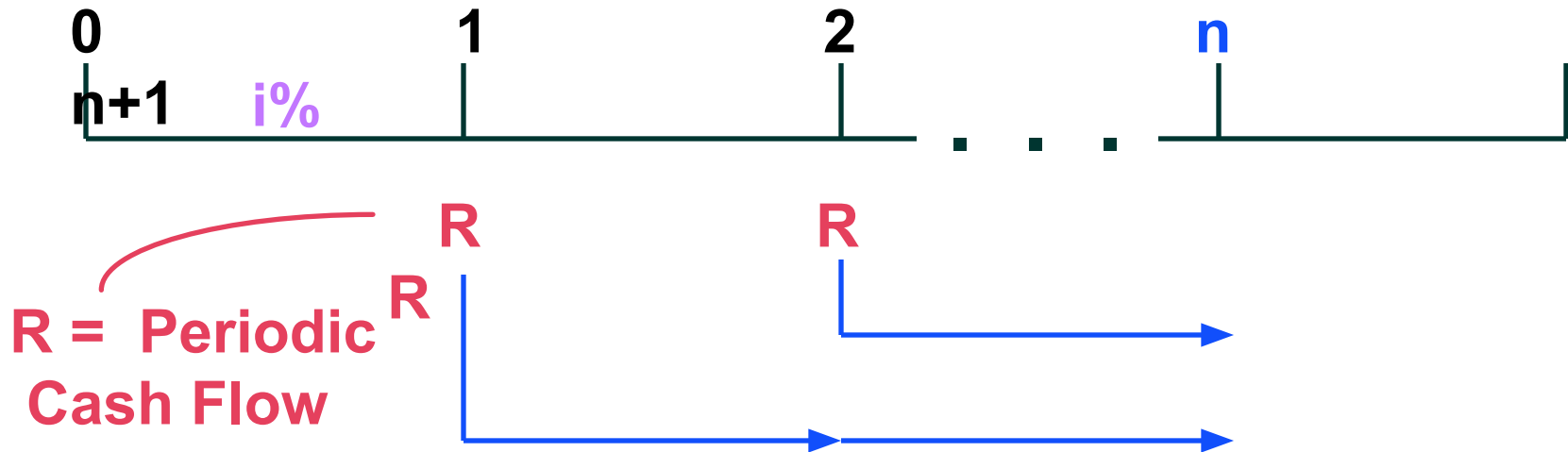
Parts of an Annuity





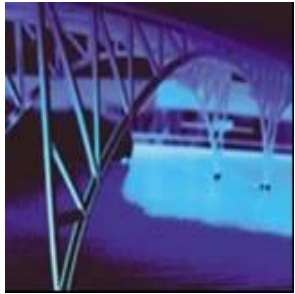
Overview of an Ordinary Annuity -- FVA

Cash flows occur at the end of the period



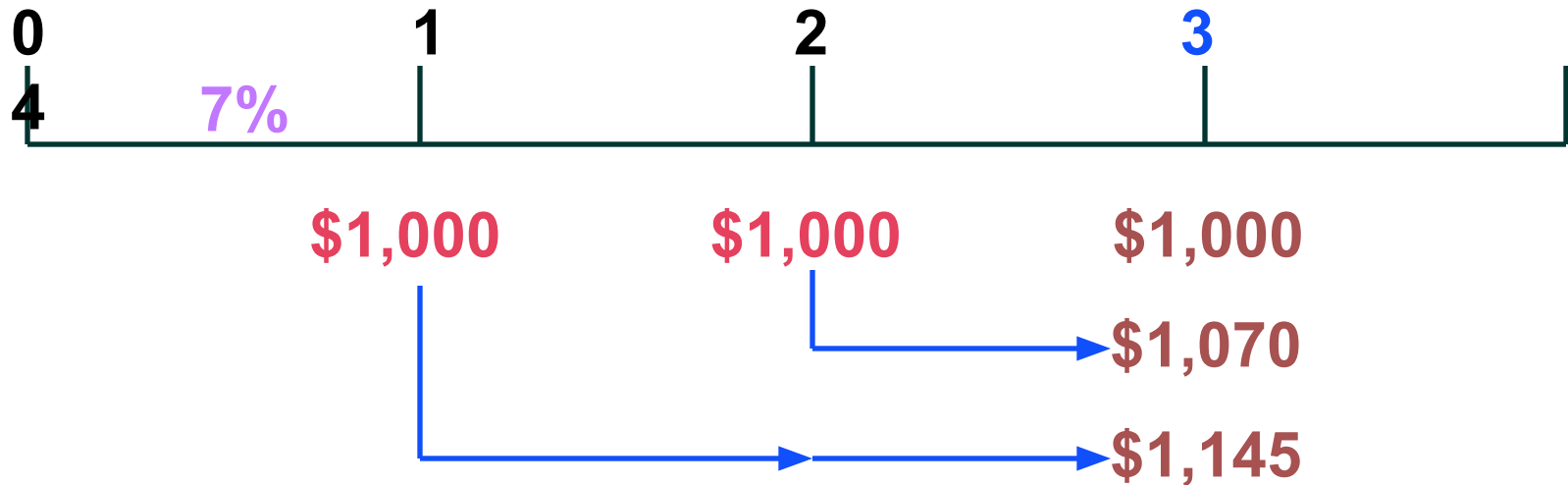
$$FVA_n = R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i)^1 + R(1+i)^0$$

FVA_n



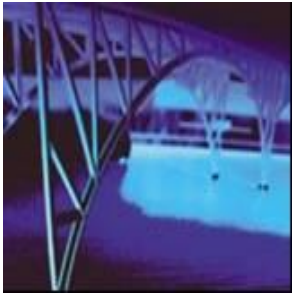
Example of an Ordinary Annuity -- FVA

Cash flows occur at the end of the period



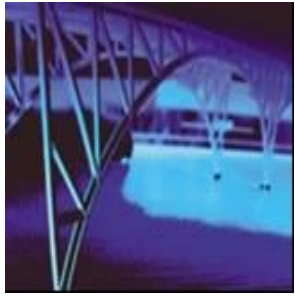
$$\begin{aligned} FVA_3 &= \$1,000(1.07)^2 + \\ &\quad \$1,000(1.07)^1 + \$1,000(1.07)^0 \\ &= \$1,145 + \$1,070 + \$1,000 \\ &= \$3,215 \end{aligned}$$

$$\underline{\$3,215} = FVA_3$$



Hint on Annuity Valuation

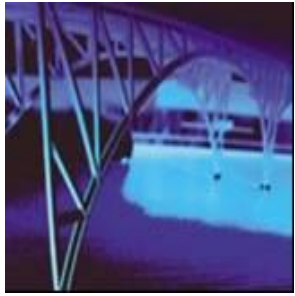
The future value of an ordinary annuity can be viewed as occurring at the end of the last cash flow period, whereas the future value of an annuity due can be viewed as occurring at the beginning of the last cash flow period.



Valuation Using Table III

$$\begin{aligned} FVA_n &= R (FVIFA_{i\%,n}) \\ &= \$1,000 (FVIFA_{7\%,3}) \\ &= \$1,000 (3.215) = \$3,215 \end{aligned} \qquad FVA_3 =$$

Period	6%	7%	8%
1	1.000	1.000	1.000
2	2.060	2.070	2.080
3	3.184	3.215	3.246
4	4.375	4.440	4.506
5	5.637	5.751	5.867



Solving the FVA Problem

Inputs	3	7	0	-1,000	
	N	I/Y	PV	PMT	FV
Compute	3,214.90				

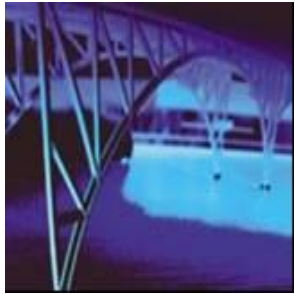
N: 3 Periods (enter as 3 year-end deposits)

I/Y: 7% interest rate per period (enter as 7 NOT .07)

PV: Not relevant in this situation (no beg value)

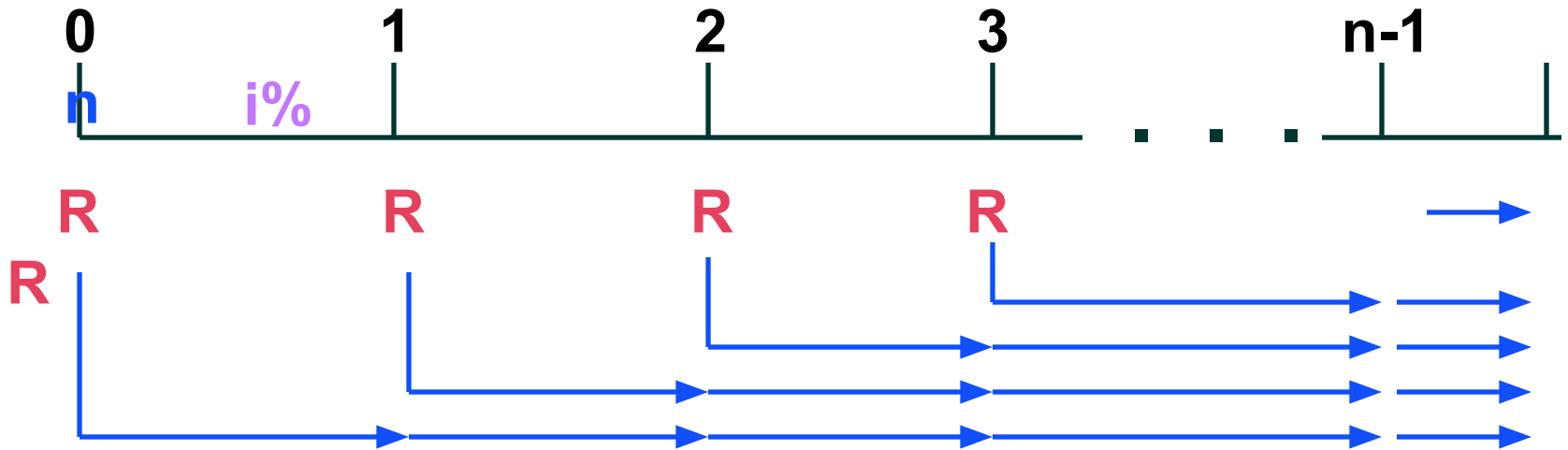
PMT: \$1,000 (negative as you deposit annually)

FV: Compute (Resulting answer is positive)



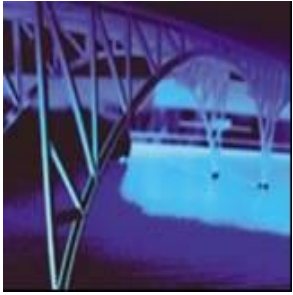
Overview View of an Annuity Due -- FVAD

Cash flows occur at the beginning of the period



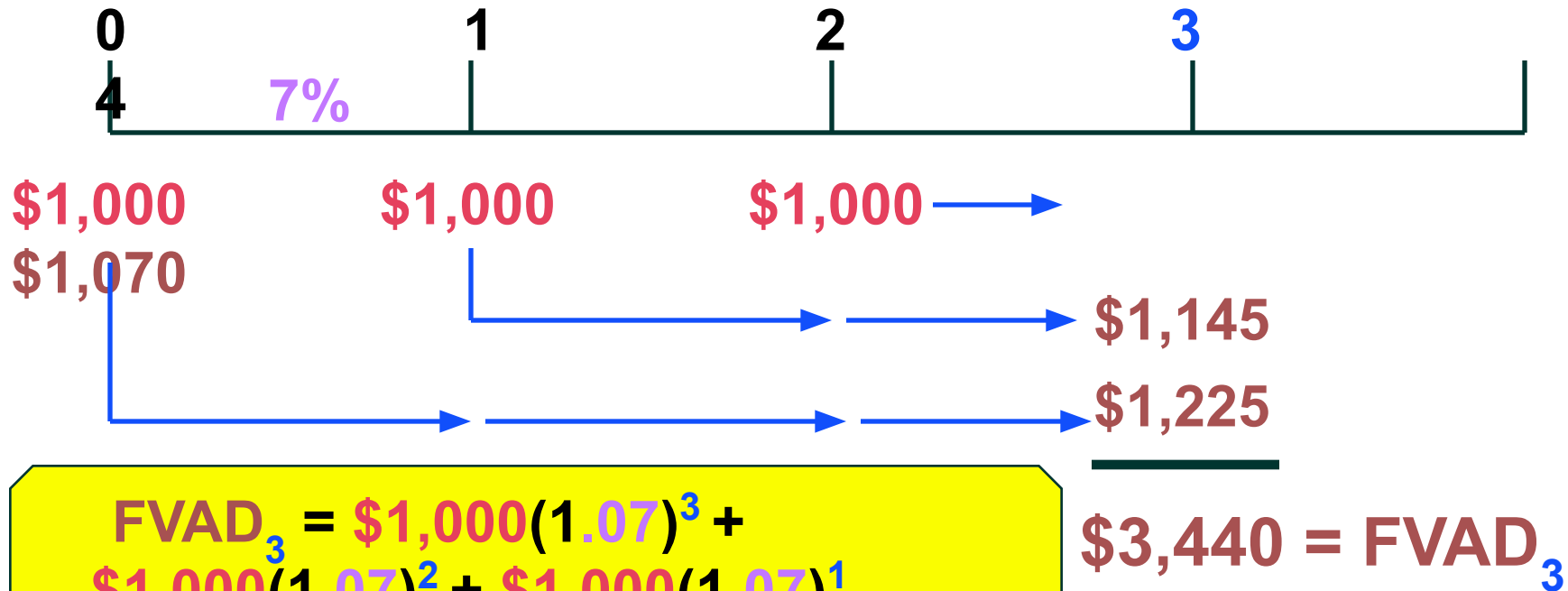
$$\begin{aligned}
 FVAD_n &= R(1+i)^n + R(1+i)^{n-1} + \\
 &\quad \dots + R(1+i)^2 + R(1+i)^1 \\
 &= FVA_n (1+i)
 \end{aligned}$$

$FVAD_n$



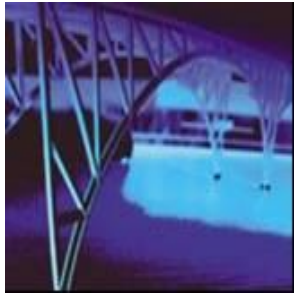
Example of an Annuity Due -- FVAD

Cash flows occur at the beginning of the period



$$\begin{aligned} \text{FVAD}_3 &= \$1,000(1.07)^3 + \\ &\quad \$1,000(1.07)^2 + \$1,000(1.07)^1 \\ &= \$1,225 + \$1,145 + \$1,070 \\ &= \$3,440 \end{aligned}$$

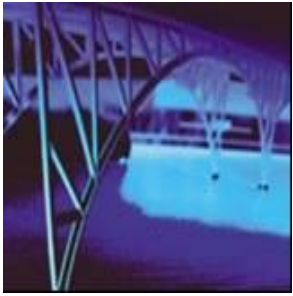
$$\underline{\$3,440} = \text{FVAD}_3$$



Valuation Using Table III

$$\begin{aligned}
 FVAD_n &= R (FVIFA_{i\%,n})(1+i) \\
 FVAD_3 &= \$1,000 (FVIFA_{7\%,3})(1.07) \\
 &= \$1,000 (3.215)(1.07) = \$3,440
 \end{aligned}$$

Period	6%	7%	8%
1	1.000	1.000	1.000
2	2.060	2.070	2.080
3	3.184	3.215	3.246
4	4.375	4.440	4.506
5	5.637	5.751	5.867



Solving the FVAD Problem

Inputs	3	7	0	-1,000	
	N	I/Y	PV	PMT	FV
Compute	3,439.94				

Complete the problem the same as an “*ordinary annuity*” problem, except you must change the calculator setting to “BGN” first. Don’t forget to change back!

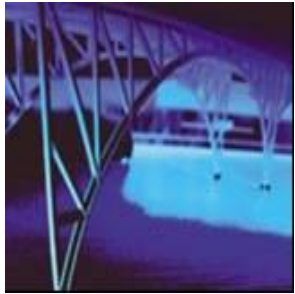
- Step 1: Press 2nd BGN keys
- Step 2: Press 2nd SET keys
- Step 3: Press 2nd QUIT keys



A horizontal line represents an array. Above the line, indices are marked: 0, 1, 2, ..., n. Below the line, the element at index i is labeled i%.



3-51



Example of an Ordinary Annuity -- PVA

Cash flows occur at the end of the period



\$1,000

\$1,000

\$1,000

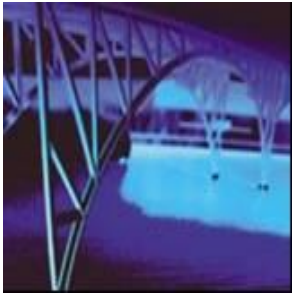
\$934.58

\$873.44

\$816.30

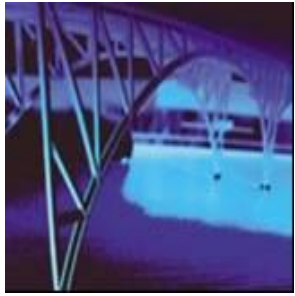
$\$2,624.32 = PVA_3$

$$\begin{aligned} PVA_3 &= \$1,000/(1.07)^1 + \\ &\quad \$1,000/(1.07)^2 + \\ &\quad \$1,000/(1.07)^3 \\ &= \$934.58 + \$873.44 + \$816.30 \\ &= \$2,624.32 \end{aligned}$$



Hint on Annuity Valuation

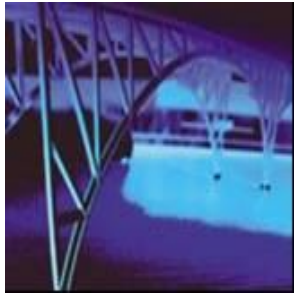
The **present value** of an **ordinary annuity** can be viewed as occurring at the **beginning** of the first cash flow period, whereas the **future value** of an **annuity due** can be viewed as occurring at the **end** of the first cash flow period.



Valuation Using Table IV

$$\begin{aligned} PVA_n &= R (PVIFA_{i\%,n}) \\ &= \$1,000 (PVIFA_{7\%,3}) \\ &= \$1,000 (2.624) = \$2,624 \end{aligned} \qquad PVA_3 =$$

Period	6%	7%	8%
1	0.943	0.935	0.926
2	1.833	1.808	1.783
3	2.673	2.624	2.577
4	3.465	3.387	3.312
5	4.212	4.100	3.993



Solving the PVA Problem

Inputs	3	7		-1,000	0
	N	I/Y	PV	PMT	FV
Compute			2,624.32		

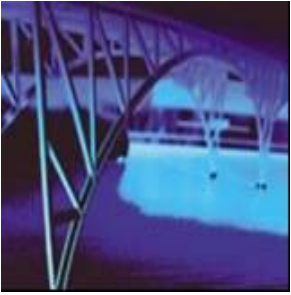
N: 3 Periods (enter as 3 year-end deposits)

I/Y: 7% interest rate per period (enter as 7 NOT .07)

PV: Compute (Resulting answer is positive)

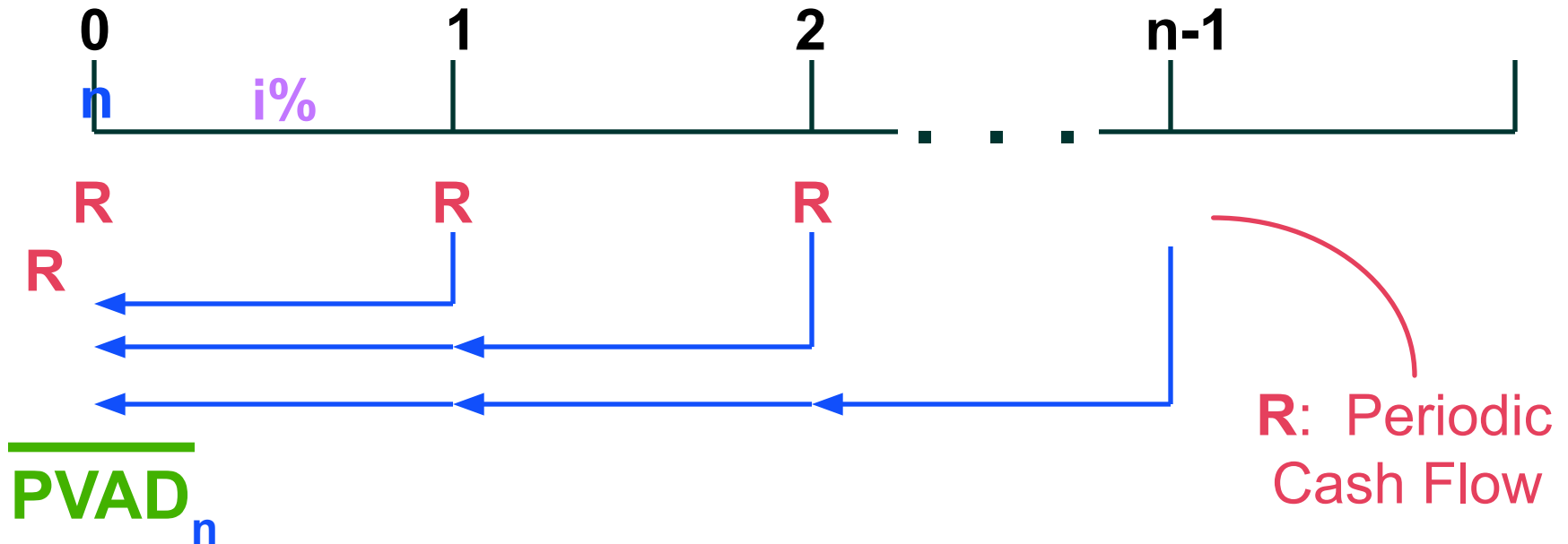
PMT: \$1,000 (negative as you deposit annually)

FV: Not relevant in this situation (no ending value)

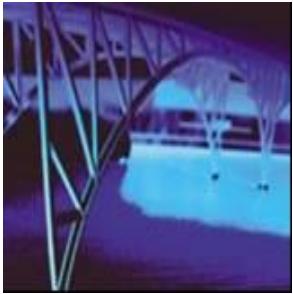


Overview of an Annuity Due -- PVAD

Cash flows occur at the beginning of the period

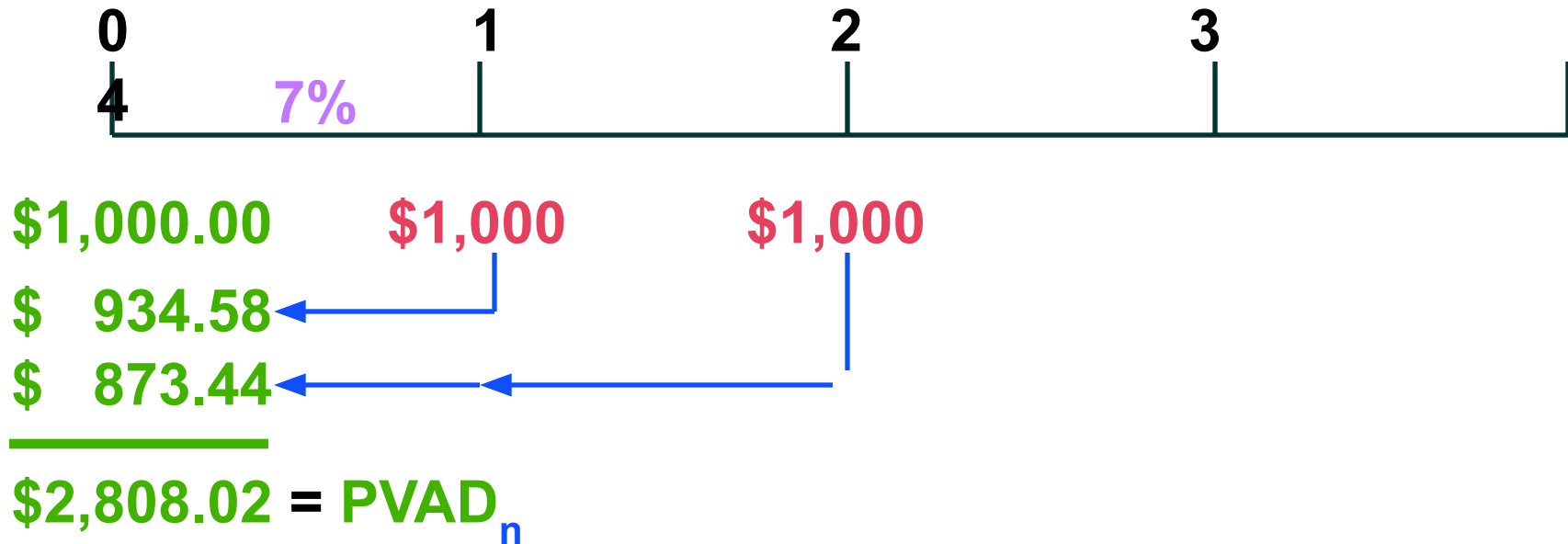


$$\begin{aligned}
 PVAD_n &= R/(1+i)^0 + R/(1+i)^1 + \dots + R/(1+i)^{n-1} \\
 &= PVA_n (1+i)
 \end{aligned}$$

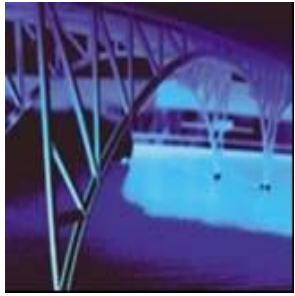


Example of an Annuity Due -- PVAD

Cash flows occur at the beginning of the period



$$\text{PVAD}_n = \$1,000/(1.07)^0 + \$1,000/(1.07)^1 + \$1,000/(1.07)^2 = \$2,808.02$$

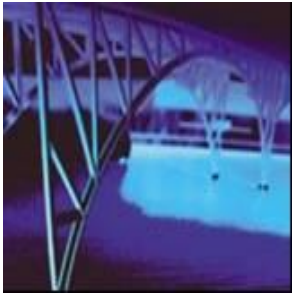


Valuation Using Table IV

$$PVAD_n = R (PVIFA_{i\%,n})(1+i)$$

$$PVAD_3 = \$1,000 (PVIFA_{7\%,3})(1.07) \\ = \$1,000 (2.624)(1.07) = \$2,808$$

Period	6%	7%	8%
1	0.943	0.935	0.926
2	1.833	1.808	1.783
3	2.673	2.624	2.577
4	3.465	3.387	3.312
5	4.212	4.100	3.993



Solving the PVAD Problem

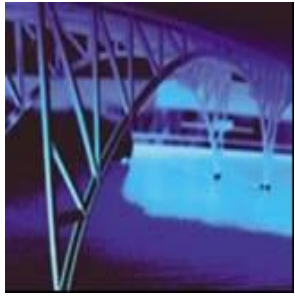
Inputs	3	7		-1,000	0
	N	I/Y	PV	PMT	FV
Compute			2,808.02		

Complete the problem the same as an “*ordinary annuity*” problem, except you must change the calculator setting to “BGN” first. Don’t forget to change back!

Step 1: Press 2nd BGN keys

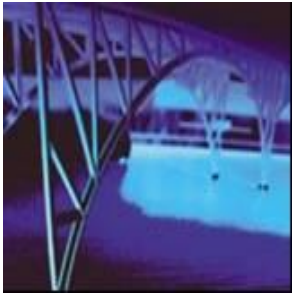
Step 2: Press 2nd SET keys

Step 3: Press 2nd QUIT keys



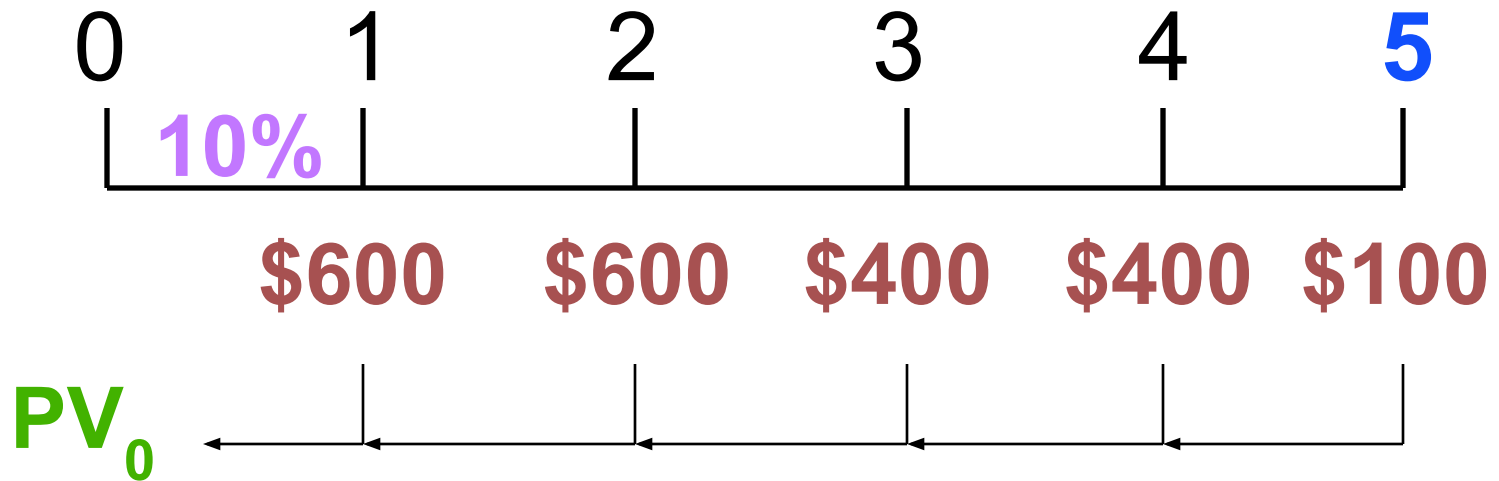
Steps to Solve Time Value of Money Problems

- 1. Read problem thoroughly**
- 2. Create a time line**
- 3. Put cash flows and arrows on time line**
- 4. Determine if it is a PV or FV problem**
- 5. Determine if solution involves a single CF, annuity stream(s), or mixed flow**
- 6. Solve the problem**
- 7. Check with financial calculator (optional)**



Mixed Flows Example

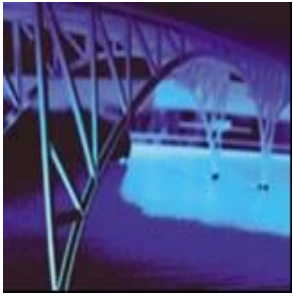
Julie Miller will receive the set of **cash flows** below. What is the **Present Value** at a discount rate of **10%**.



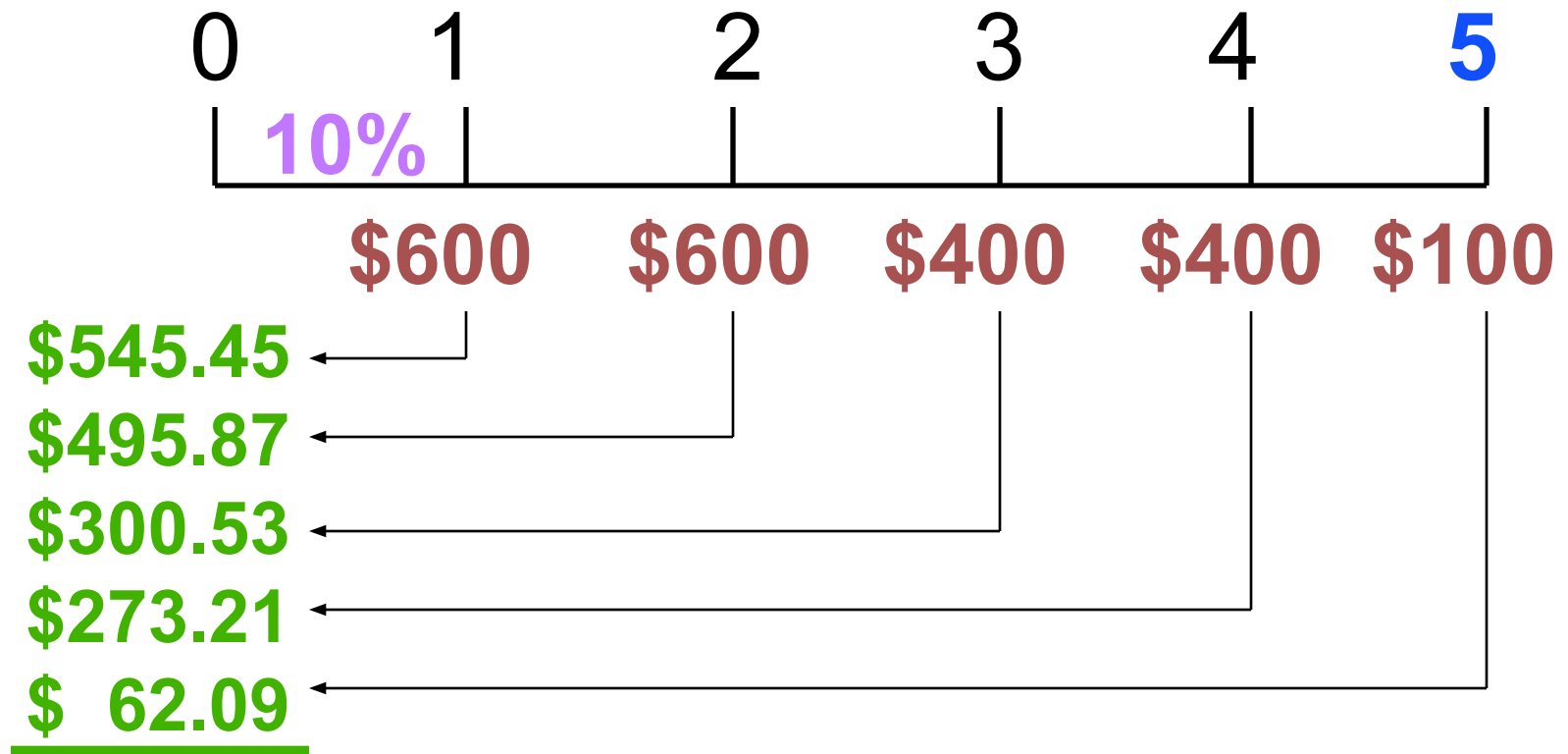


How to Solve?

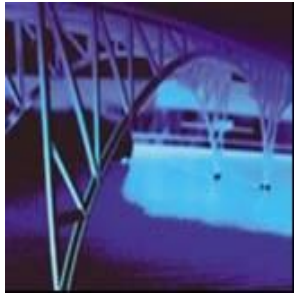
1. Solve a “**piece-at-a-time**” by discounting each **piece** back to $t=0$.
2. Solve a “**group-at-a-time**” by first breaking problem into groups of annuity streams and any single cash flow groups. Then discount each **group** back to $t=0$.



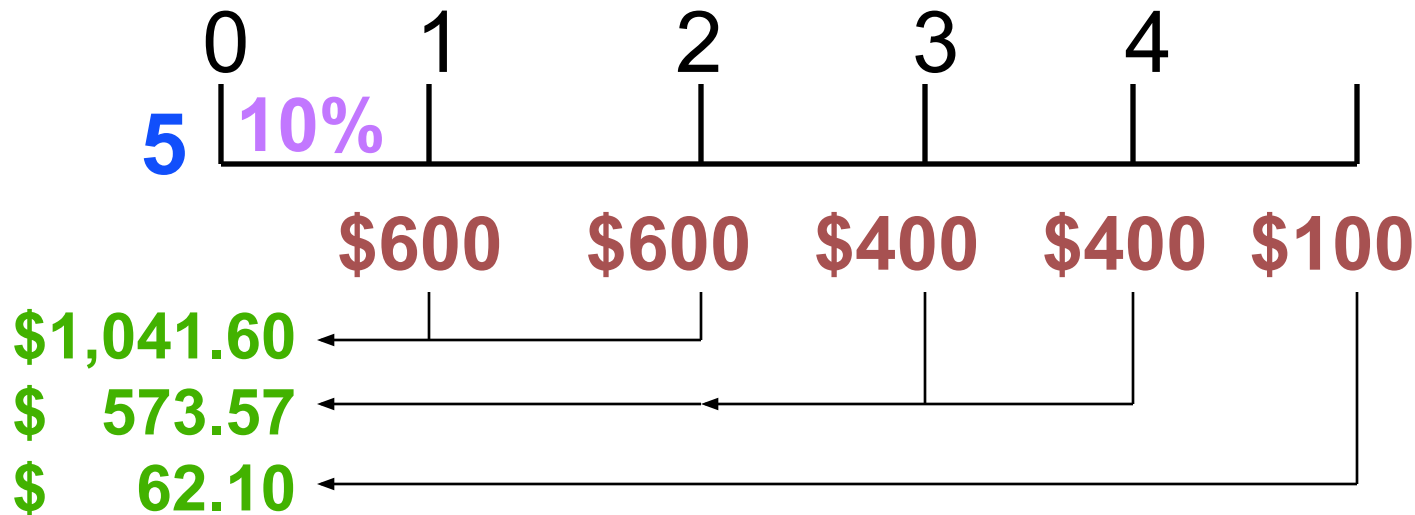
“Piece-At-A-Time”



\$1677.15 = PV_0 of the Mixed Flow

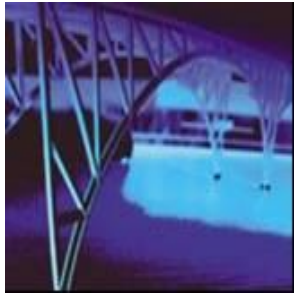


“Group-At-A-Time” (#1)

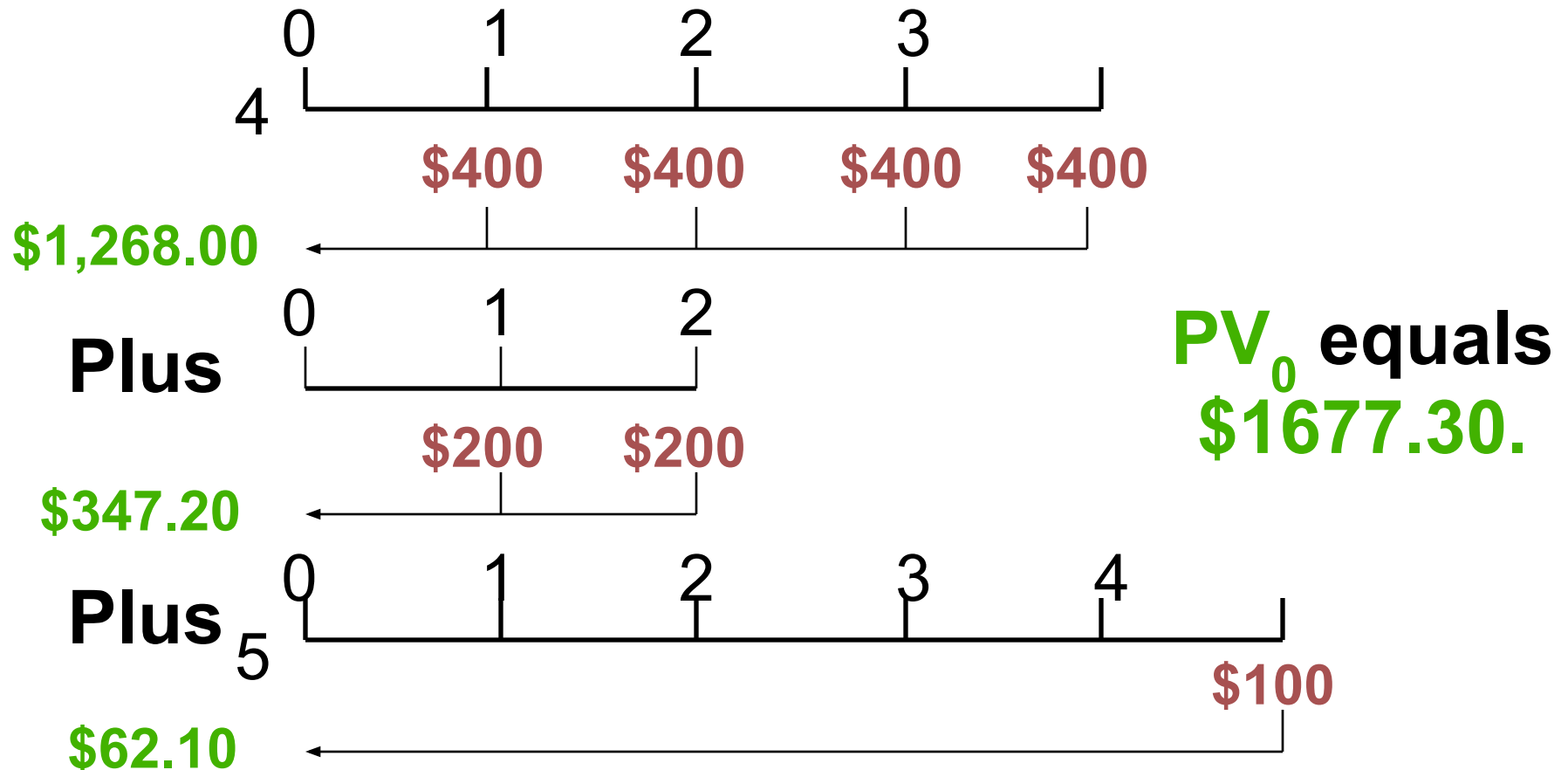


\$1,677.27 = PV_0 of Mixed Flow [Using Tables]

$$\begin{aligned}
 \$600(\text{PVIFA}_{10\%,2}) &= \$600(1.736) = \$1,041.60 \\
 \$400(\text{PVIFA}_{10\%,2})(\text{PVIF}_{10\%,2}) &= \$400(1.736)(0.826) = \$573.57 \\
 \$100(\text{PVIF}_{10\%,5}) &= \$100(0.621) = \$62.10
 \end{aligned}$$

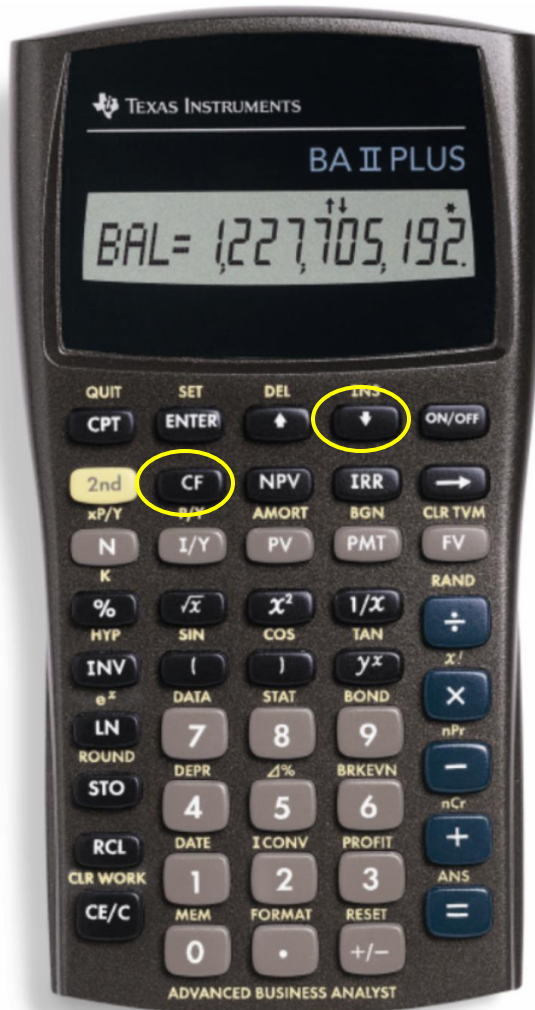


“Group-At-A-Time” (#2)

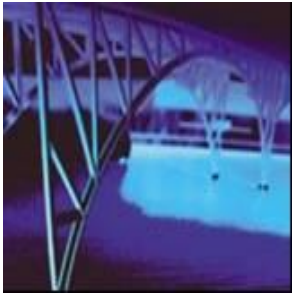




Solving the Mixed Flows Problem using CF Registry



- Use the highlighted key for starting the process of solving a mixed cash flow problem
- Press the CF key and down arrow key through a few of the keys as you look at the definitions on the *next slide*



Solving the Mixed Flows Problem using CF Registry

Defining the calculator variables:

For CF_0 : This is ALWAYS the cash flow occurring at time $t=0$ (usually 0 for these problems)

For C_{nn} :* This is the cash flow SIZE of the n th group of cash flows. Note that a “group” may only contain a single cash flow (e.g., \$351.76).


















For F_{nn} :* This is the cash flow FREQUENCY of the n th group of cash flows. Note that this is always a positive whole number (e.g., 1, 2, 20, etc.).

* nn represents the n th cash flow or frequency. Thus, the first cash flow is C_{01} , while the tenth cash flow is C_{10} .



Solving the Mixed Flows Problem using CF Registry

Steps in the Process

- Step 1: Press CF  key
- Step 2: Press 2nd CLR Work  keys
- Step 3: For CF0 Press  Enter   keys
- Step 4: For C01 Press  Enter   keys
- Step 5: For F01 Press  Enter   keys
- Step 6: For C02 Press  Enter   keys
- Step 7: For F02 Press  Enter   keys



Solving the Mixed Flows Problem using CF Registry

Steps in the Process

Step 8: For C03 Press **100** **Enter** **↓** keys

Step 9: For F03 Press **1** **Enter** **↓** keys

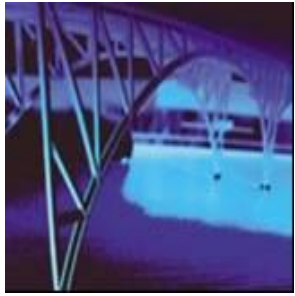
Step 10: Press **↓** keys

Step 11: Press **NPV** key

Step 12: For I=, Enter 10 **Enter** **↓** keys

Step 13: Press **CPT** key

Result: **Present Value** = **\$1,677.15**



Frequency of Compounding

General Formula:

$$FV_n = PV_0(1 + [i/m])^{mn}$$

n : Number of Years

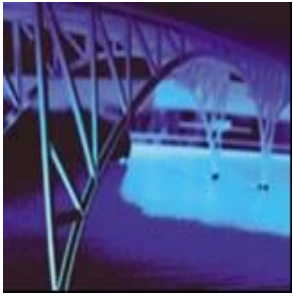
m :

Compounding Periods per Year **i :**

Annual Interest Rate
the end of Year n

$FV_{n,m}$: FV at

PV_0 : PV of the Cash Flow today

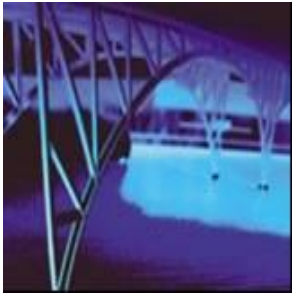


Impact of Frequency

Julie Miller has **\$1,000** to invest for **2 Years** at an annual interest rate of **12%**.

$$\text{Annual } FV_2 = 1,000(1 + [.12/1])^{(1)(2)} \\ = 1,254.40$$

$$\text{Semi } FV_2 = 1,000(1 + [.12/2])^{(2)(2)} \\ = 1,262.48$$

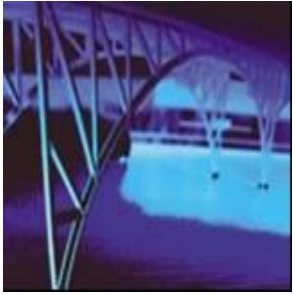


Impact of Frequency

Qrtly $FV_2 = 1,000(1 + [.12/4])^{(4)(2)}$
 $= 1,266.77$

Monthly $FV_2 = 1,000(1 + [.12/12])^{(12)(2)}$
 $= 1,269.73$

Daily $FV_2 = 1,000(1 + [.12/365])^{(365)(2)}$
 $= 1,271.20$

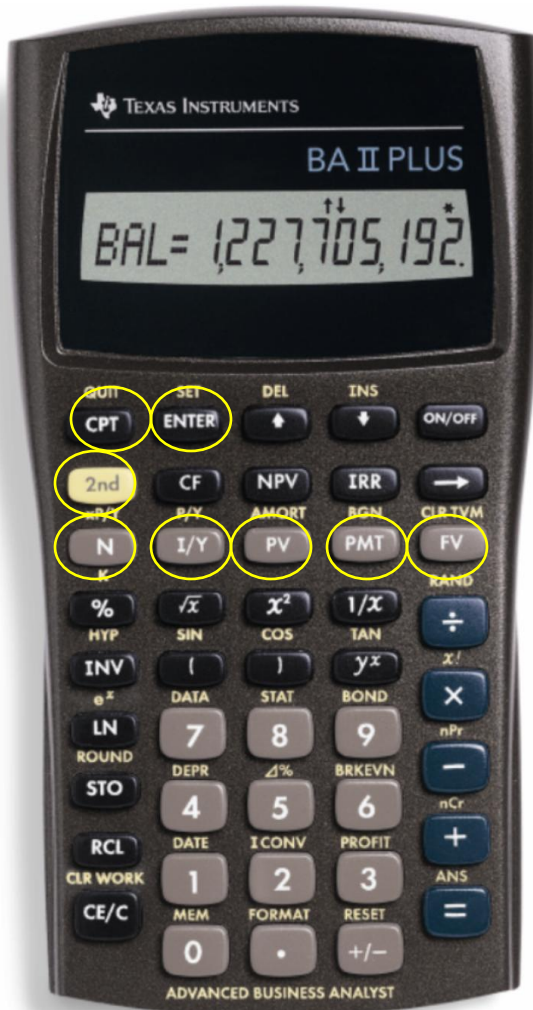


Solving the Frequency Problem (Quarterly)

Inputs	2(4)	12/4	-1,000	0	
	N	I/Y	PV	PMT	FV
Compute	1266.77				

The result indicates that a **\$1,000** investment that earns a **12%** annual rate compounded quarterly for **2 years** will earn a future value of **\$1,266.77**.

Solving the Frequency Problem (Quarterly Altern.)



Press:

2nd P/Y 4 ENTER

2nd QUIT

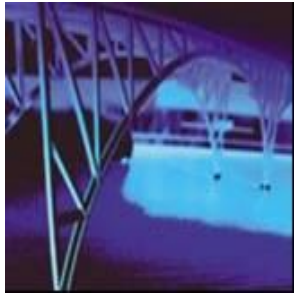
12 I/Y

-1000 PV

0 PMT

2 2nd xP/Y N

CPT FV

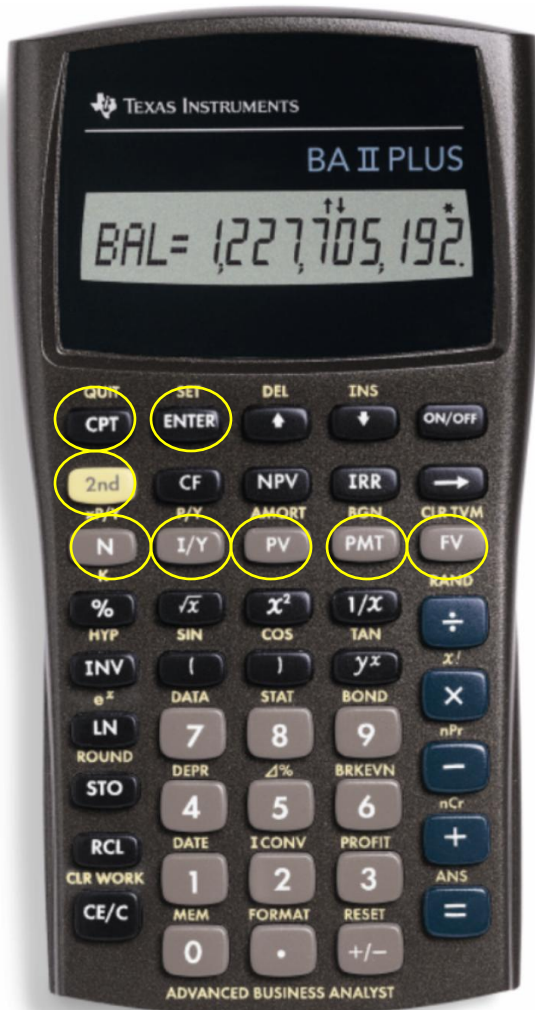


Solving the Frequency Problem (Daily)

Inputs	2(365)	12/365	-1,000	0	
	N	I/Y	PV	PMT	FV
Compute	1271.20				

The result indicates that a **\$1,000** investment that earns a **12%** annual rate compounded daily for **2 years** will earn a future value of **\$1,271.20**.

Solving the Frequency Problem (Daily Alternative)



Press:

2nd P/Y 365 ENTER

2nd QUIT

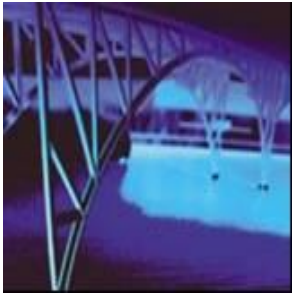
12 I/Y

-1000 PV

0 PMT

2 2nd xP/Y N

CPT FV

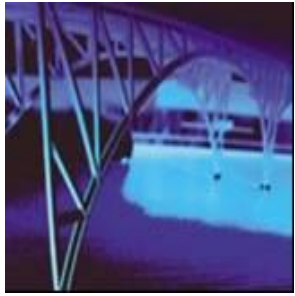


Effective Annual Interest Rate

Effective Annual Interest Rate

The actual rate of interest earned (paid) after adjusting the *nominal rate* for factors such as the number of **compounding periods per year**.

$$(1 + [i / m])^m - 1$$



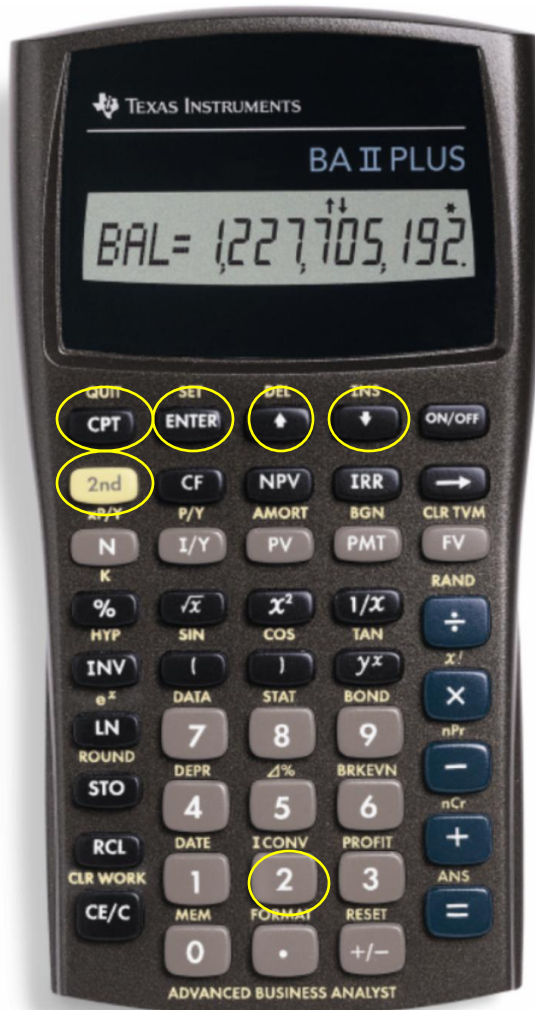
BWs Effective Annual Interest Rate

Basket Wonders (BW) has a \$1,000 CD at the bank. The interest rate is **6% compounded quarterly** for 1 year. What is the Effective Annual Interest Rate (**EAR**)?

$$\begin{aligned} \text{EAR} &= (1 + 6\% / 4)^4 - 1 &= \\ 1.0614 - 1 &= .0614 \text{ or } 6.14\%! \end{aligned}$$

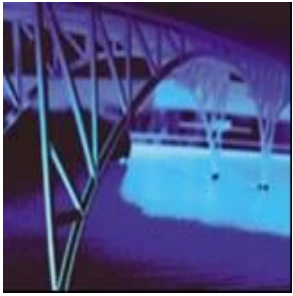


Converting to an EAR



Press:

2 nd	I Conv
6	ENTER
↓	↓
4	ENTER
↑	CPT
2 nd	QUIT



Steps to Amortizing a Loan

1. Calculate the **payment per period**.
2. Determine the **interest** in Period t .
(**Loan Balance** at $t-1$) \times ($i\% / m$)
3. Compute **principal payment** in Period t .
(**Payment** - **Interest** from Step 2)
4. Determine ending balance in Period t .
(**Balance** - **principal payment** from Step 3)
5. Start again at Step 2 and repeat.



Amortizing a Loan Example

Julie Miller is borrowing **\$10,000** at a compound annual interest rate of **12%**.
Amortize the loan if **annual payments** are made for **5 years**.

Step 1: Payment

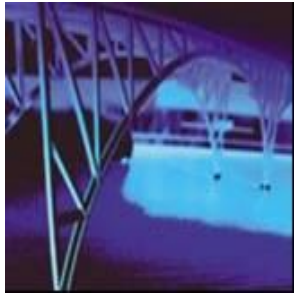
$$\begin{aligned}PV_0 &= R (PVIFA_{i\%,n}) \\ \$10,000 &= R (PVIFA_{12\%,5}) \\ \$10,000 &= R (3.605) \\ R &= \$10,000 / 3.605 = \$2,774\end{aligned}$$



Amortizing a Loan Example

End of Year	Payment	Interest	Principal	Ending Balance
0	---	---	---	\$10,000
1	\$2,774	\$1,200	\$1,574	8,426
2	2,774	1,011	1,763	6,663
3	2,774	800	1,974	4,689
4	2,774	563	2,211	2,478
5	2,775	297	2,478	0
	\$13,871	\$3,871	\$10,000	

[Last Payment Slightly Higher Due to Rounding]

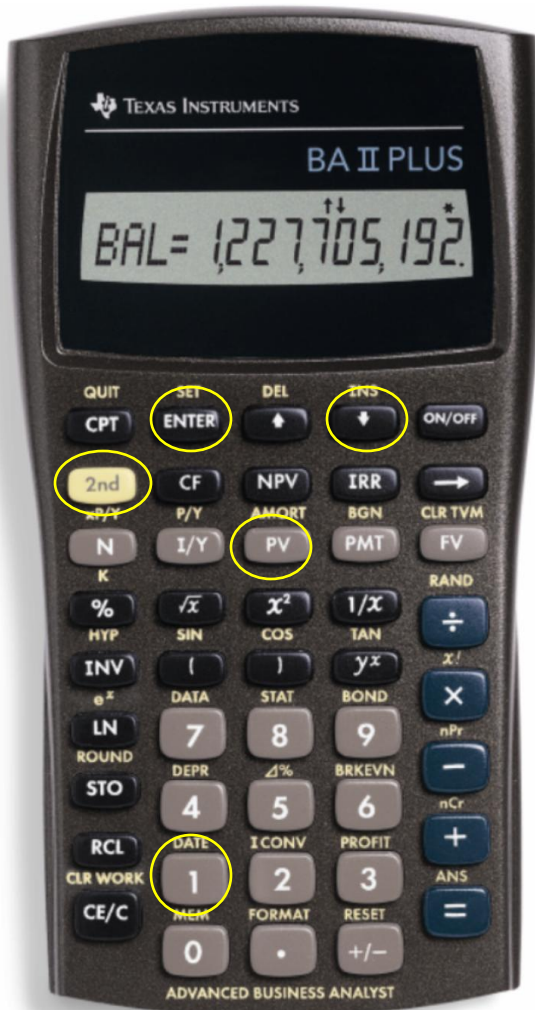


Solving for the Payment

Inputs	5	12	10,000		0
	N	I/Y	PV	PMT	FV
Compute				-2774.10	

The result indicates that a **\$10,000** loan that costs **12%** annually for **5 years** and will be **completely paid off** at that time will require **\$2,774.10** annual payments.

Using the Amortization Functions of the Calculator



Press:

2 nd	Amort
1	ENTER
1	ENTER

Results:

BAL = 8,425.90*



PRN = -1,574.10*



INT = -1,200.00*

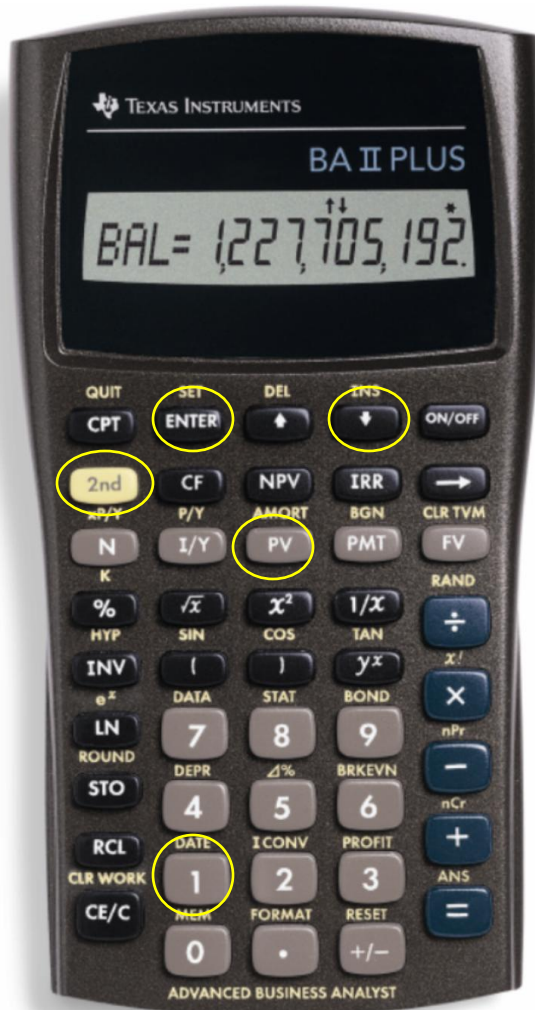


Year 1 information only

*Note: Compare to 3-82



Using the Amortization Functions of the Calculator



Press:

2nd	Amort
2	ENTER
2	ENTER

Results:

BAL = 6,662.91*



PRN = -1,763.99*



INT = -1,011.11*

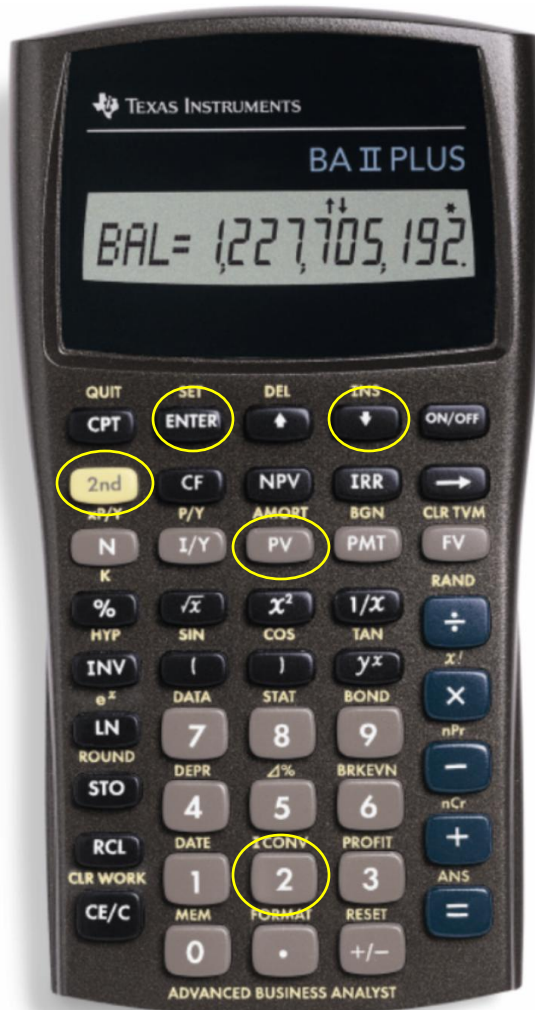


Year 2 information only

***Note: Compare to 3-82**



Using the Amortization Functions of the Calculator



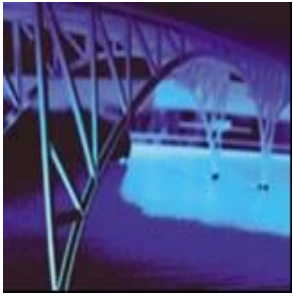
Press:

2nd	Amort
1	ENTER
5	ENTER

Results:

BAL =	0.00	↓
PRN =	-10,000.00	↓
INT =	-3,870.49	↓

**Entire 5 Years of loan information
(see the total line of 3-82)**



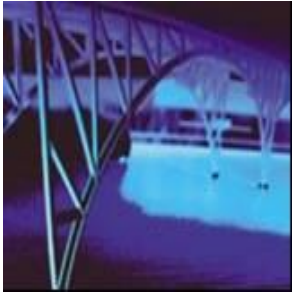
Usefulness of Amortization

1. Determine Interest Expense --

Interest expenses may reduce taxable income of the firm.

2. Calculate Debt Outstanding --

The quantity of outstanding debt may be used in financing the day-to-day activities of the firm.



- **Thank you**